Online Appendix for
Misallocation in the Market for Inputs:
Enforcement and the Organization of Production

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June 2, 2020

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A Data Appendix

A.1 Data Sources and Variable Definitions

- *Plant-level data:* Our plant data is India's Annual Survey of Industries (ASI), published by the Central Statistics Office, Ministry of Statistics and Program Implementation. The data is at annual frequency, each reporting year starts on April 1st and ends on March 31st. Our data covers the years 2000/01 to 2012/13. The input and output product codes for 2008/09 and 2009/10 are different from the ones for earlier years; we created a concordance using the product names (which often match perfectly), and concord the few remaining ones by hand. The years 2010/11 to 2012/13 use the NPCMS product classification, which we convert to ASIC 2007/08 product codes using the concordance published by the Ministry.

- *Total cost:* Sum of the user cost of capital, the total wage bill, energy, services, and materials inputs. Total cost is set to be missing if and only if the user cost of capital, the wage bill, or total materials are missing. The user cost of capital is constructed using the perpetual inventory method as in the Appendix of Greenstreet (2007), using depreciation rates of 0%, 5%, 10%, 20%, and 40% for land, buildings, machinery, transportation equipment, and computers & software, respectively. Capital deflators are from the Ministry's wholesale price index, and the nominal interest rate is the India Bank Lending Rate, from the IMF's International Financial Statistics (on average about 11%).

- *Materials expenditure in total cost:* (as used in Table I and subsequent tables) Total expenditure on intermediate inputs which are associated with a product code (that excludes services and most energy inputs) divided by total cost (see above).

- *Pending High Court cases:* From Daksh India (www.dakshindia.org). Daksh collects and updates pending cases by scraping High Court websites. Cases were retrieved on 11 June 2017. To eliminate biases resulting from possible delays in the digitization, we exclude all cases that were filed after 1/1/2017. We divide cases into civil and criminal based on state-specific case type codes (which are part of the case identifiers), and a correspondence between case types and whether they are civil or criminal cases (from High Courts, where available).

- *Rauch classification of goods*: From James Rauch's website, for 5-digit SITC codes. Concorded from SITC codes to ASIC via the SITC-CPC concordance from UNSTATS, and the NPCMS-ASIC concordance from the Indian Ministry of Statistics (NPCMS is based on CPC codes).

- *Dependence on relationship-specific inputs, by industry:* (as used in Table I) Following Nunn (2007): total expenditure of single-product plants in an industry on relationship-specific inputs (according to the concorded Rauch classification), by 3-digit industry, divided by total expenditures on intermediate inputs that are associated with a 5-digit product code (which excludes services and most energy intermediate inputs).

- *Gross domestic product per capita, by district:* District domestic product was assembled from various state government reports, for the year 2005 (to maximize coverage). Missing for Goa and Gujarat and some union territories, and for some individual districts in the other states. Population data from the 2001 and 2011 Census of India, interpolated to 2005 assuming a constant population growth rate in each district. Whenever district domestic product per capita was unavailable, we used gross state domestic product per capita, as reported by the Ministry of Statistics and Program Implementation.

- *Vertical Span:* See Appendix B. Due to the change in product classification from ASIC to NPCMS after 2010, we construct vertical distance only using the pre-2011 years.

- *Trust:* Fraction of respondents that answer "Most people can be trusted" in the World Value Survey's trust question: "Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?" Data from waves 4, 5, and 6 of the World Values Survey, except for Himachal Pradesh, Puducherry, and Uttarakhand, which are only available in wave 6. Data is missing for Goa and the UT's, except for Delhi and Puducherry.

- *Language:* Data on population by mother tongue in each state, from the 2001 Census of India, as published by the Office of the Registrar General and Census Commissioner.
- *Caste:* Data on caste from the 2014 round of CMIE's Consumer Pyramids Survey (as made available by ICPSR), covering all states of India except the northeastern states and the UT's of Andaman and Nicobar, Lakshadweep, Dadra and Nagar Haveli and Daman and Diu. Observations have been weighted to be representative at the level of a "homogeneous region", which is "a set of neighbouring districts that have similar agro-climatic conditions, urbanisation levels and female literacy" (CPS 2014 User guide). Data covers 364 castes and caste categories. Herfindahls are constructed at the level of a homogeneous region, and mapped to districts.[1]
- *Corruption:* Number of self-reported bribes per 1,000 inhabitants. Data on bribes is the full history of 35,391 self-reported bribes paid from IPaidABribe.com (as of 03/28/2018), which we aggregate at the state level. Population by state is from the 2011 Census of India.
- *Capital Intensity:* Average plant-level cost share of capital inputs (user cost of capital, see above), by 5-digit industry.
- *Wage Premium:* Average plant-level wage bill divided by the total number of man-days worked, by 5-digit industry.
- *Contract Labor Share:* Average plant-level cost share of contract (non-permanent) workers in the total wage bill, by 5-digit industry.
- *Upstreamness:* Upstreamness as defined by Antràs et al. (2012) of 3-digit industries, calculated by year and averaged across years. Some goods show up mostly as inputs and not as outputs because as outputs they fall outside of the scope of the ASI (chiefly agricultural and mining goods); for these inputs total observed intermediate consumption exceeds total observed production by plants in our sample. We set total production of these goods equal to total intermediate consumption (hence assuming zero sales to final goods consumers).[2] The resulting variable looks very similar to those constructed by Antràs et al. (2012) for industrialized countries.
- *Tradability:* Weighted average freight rates of industry's inputs, by 5-digit industry (using only single-product plants). Freight rates are the "average trade-weighted freight rates" by 2-digit SITC codes, from Hummels (1999), concorded to ASIC codes using the SITC-ASIC concordance that we used for the Rauch classification as well.
- *Household consumption shares HH_ω:* Define total net production as the total production of ω (from the ASI, pooled across all years within each state, with each plant-year observation weighted by the inverse of the number of times the plant shows up in the ASI), minus the total consumption of ω as intermediate inputs by ASI firms (constructed and weighted analogously). If total net production of a good is negative, we set it equal to zero. The value of HH_ω is then the fraction of total net production of ω is the sum of total net production of all goods $\omega' \in \Omega$.
- *Recipe revenue shares $R_{\omega\rho}$:* Share of sales of ω using recipe ρ in total sales of ω. The sales of ω using ρ of a single-product plant j are the sales of ω by j multiplied by the probability that j produces using ρ (equation (3)). To construct the sales of ω using ρ of a multi-product plant j, we choose plant-specific recipe shares to minimize the Euclidean distance of the plant's vector of observed cost shares from the weighted average of the recipes' mean cost shares $\overline{m}_{\rho\omega}$, where the weights are the plant-specific recipe shares (subject to the constraint that weights have to sum up to one for each product ω). To construct $R_{\omega\rho}$, we weigh all plant-year observations by the inverse of the number of times the plant shows up in the ASI, pooling across all states and years.

[1] We are grateful to Renuka Sané and CMIE for helping us get a description of the homogeneous regions.

[2] While this may make these goods look more upstream than they actually are, the biases incurred are likely to be small: minerals are usually not directly sold to households; agricultural goods are either very upstream in the value chain of processed foods, or directly sold to households, but do not appear "in the middle" of the value chain.

A.2 Sample

For the linear regressions in Section 2, the sample consists of all plants that are reported as operating, produce a single 5-digit product, and have materials shares in sales strictly between zero and two. We also drop the few observations from Sikkim and the Seven Sister States in North East India (Assam, Meghalaya, Manipur, Mizoram, Nagaland, and Tripura; Arunachal Pradesh is not covered by the ASI) because they have a different sampling methodology in the ASI, and Jammu and Kashmir, because coverage of its court cases is inadequate, and because many federal laws do not apply to it due to its special status within the union.

For the structural estimation, we also remove observations where the shares of relationship-specific materials, homogeneous materials, or labor in sales exceed two, and observations where sales or non-materials expenditures are non-positive. To construct the households consumption shares HH_ω and the recipe sales shares $R_{\omega\rho}$ in the counterfactual, we weigh each plant-year observation by the inverse of the number of times the plant shows up in our sample.

The regression to estimate ζ (Table VI) is the only one where we identify parameters from time variation. Our sample to construct the expenditure aggregates consists of all census plants (which are surveyed every year). We restrict the sample in this way to eliminate fluctuations in the expenditure ratios that arise artificially from changes in the sample.

A.3 Details on High Court and State Creation

Table A.1 Details on High Court Creation

Name	Jurisdiction over States/UT's	Year founded	Created with State	Notes on High Court creation	Reasons for State creation
Allahabad High Court	Uttar Pradesh	1866	no	Created as HC of Judicature of the Northwestern Provinces by the Indian High Courts Act 1861.	
High Court of Judicature at Hyderabad	Andhra Pradesh, Telangana	1956	yes	Created when Andhra Pradesh was created as part of the State Reorganization Act 1956	Creation of Andhra Pradesh was triggered by the independence movement of the Telugu-speaking population of Madras Presidency.
Mumbai High Court	Goa, Dadra and Nagar Haveli,Daman and Diu, Maharashtra	1862	no	Created by the Indian High Courts Act 1861.	
Kolkata High Court	Andaman and Nicobar Islands,West Bengal	1862	no	Created by the Indian High Courts Act 1861.	
Chhattisgarh High Court	Chhattisgarh	2000	yes	Created when Chhattisgarh state was carved out of Mandhya Pradesh in 2000 (Madhya Pradesh Reorganisation Act)	Chhattisgarh was a separate division in the Central Provinces under British rule. Demand for a separate state goes back to the 1920s.
Delhi High Court	Delhi	1966	no	At the time of independence, Punjab HC had jurisdiction for Delhi. With the State Reorganisation Act 1956, Punjab merged with Pepsu. Given Delhi's importance as capital, it was decided that they should have their own HC.	
Gauhati High Court	Arunachal Pradesh, Assam, Mizoram, Nagaland *	1948	yes	Created as HC of Assam in 1948, with the Indian constitution; renamed Gauhati HC in 1971 with the North East Areas Reorganization Act. Lost jurisdiction over Meghalaya, Manipur, and Tripura in 2013	
Gujarat High Court	Gujarat	1960	yes	Created when Gujarat split from Bombay State with the Bombay Reorganisation Act 1960.	Gujarat was created following the demand of Gujarati-speaking people for their own state (Mahagujarat movement).
Himachal Pradesh High Court	Himachal Pradesh	1971	yes	Created with Himachal Pradesh becoming a state (and therefore should have a separate HC under the Indian constitution)	
Jammu and Kashmir High Court	Jammu and Kashmir	1928	no	Created by the Maharaja in 1928. Special status: Laws passed by the Indian parliament generally do not apply to J&K (except foreign policy, communication, defense).	
Jharkhand High Court	Jharkhand	2000	yes	Created when Jharkhand state was carved out of Bihar in 2000 (Bihar Reorganisation Act)	Jharkhand was richer in natural resources than the rest of Bihar; Jharkhand Mukti Morcha independence movement, and political considerations by ruling parties.

Name	Jurisdiction over States/UT's	Year founded	Created with State	Notes on High Court creation	Reasons for State creation
Karnataka High Court	Karnataka	1884	no	Founded by the British as the Chief Court of Mysore in 1884.	
Kerala High Court	Kerala, Lakshadweep	1956	yes	Created when Kerala was created as part of the State Reorganization Act 1956	Idea of Kerala was to combine Malayalam-speaking regions.
Madhya Pradesh High Court	Madhya Pradesh	1936	no	Established as Nagpur High Court by King George V through a Letters Patent on 2 Jan 1936. Moved to its present location at Jabalpur with the State Reorganization Act 1956.	
Chennai High Court	Puducherry, Tamil Nadu	1862	no	Created by the Indian High Courts Act 1861.	
Odisha High Court	Odisha/Orissa	1948	no	Orissa was split off from Bihar in 1936, but did not get its own high court until the drafting of the Indian constitution (1948).	
Patna High Court	Bihar	1916	yes	Created when Bihar and Orissa were split off from Bengal Presidency.	Bengal nationalism and the undoing of the 1905 Partition of Bengal.
Punjab and Haryana High Court	Chandigarh, Haryana, Punjab	1947	yes	Created with the independence of India in 1947 (former HC of Punjab in Lahore was mostly relevant for modern-day Pakistan)	
Rajasthan High Court	Rajasthan	1949	yes	Created with the foundation of Rajasthan (1948-1950)	
Sikkim High Court	Sikkim	1955	no	Established by the Maharaja of Sikkim in 1955, became an Indian High Court in 1975 when Sikkim joined India and the monarchy was abolished	
Uttarakhand High Court	Uttarakhand	2000	yes	Created when Uttaranchal was carved out of Uttar Pradesh	Uttarakhand Kranti Dal independence movement.
High Court of Mumbai, Goa Bench	Goa, Daman and Diu, Dadra and Nagar Haveli	1982	no	Prior to the HC, a Judicial Commissioner's court existed in Goa. HC was established to safeguard the judge's independence (see Bombay HC at Goa website).	
Manipur High Court	Manipur	2013	no	Parties in Manipur demanded their own high court	
Meghalaya High Court	Meghalaya	2013	no	Parties in Meghalaya demanded their own high court	
High Court Of Tripura	Tripura	2013	no	Parties in Tripura demanded their own high court	

B Vertical Span and Vertical Distance Measures

B.1 Definition

Let $X_{j\omega'}$ be the expenditure of plant j on $\omega' \in \Omega$, then define for products $\omega, \omega' \in \Omega$, and a set $B \subset \Omega$

$$\beta^B_{\omega\omega'} = \frac{\sum_{j \in J_\omega} X_{j\omega'}}{\sum_{j \in J_\omega} \sum_{\omega'' \in \Omega \setminus B} X_{j\omega''}}$$

if $\omega' \notin B$, and $\beta^B_{\omega\omega'} = 0$ otherwise. $\beta^B_{\omega\omega'}$ is the share of ω' in industry ω's materials basket that excludes inputs from B.

Denote by $A^n_{\omega\omega'}$ the set of $(n+1)$-tuples $(\omega^{(0)}, \omega^{(1)}, \ldots, \omega^{(n)}) \in \Omega^{n+1}$ such that

$$\omega^{(0)} = \omega, \quad \omega^{(n)} = \omega', \tag{1}$$

$$\omega^{(i)} \neq \omega^{(j)} \quad \forall (i,j) \in \{0, \ldots, n\}^2, i \neq j. \tag{2}$$

Intuitively, $A^n_{\omega\omega'}$ is the set of all possible non-circular product chains of length n between ω and ω'. Then let

$$\delta_{\omega\omega'} = \sum_{n=1}^{\infty} \sum_{a \in A^n_{\omega\omega'}} \frac{\lambda(a)}{\sum_{n'=1}^{\infty} \sum_{a' \in A^{n'}_{\omega\omega'}} \lambda(a')} \cdot n$$

where

$$\lambda : A^n_{\omega\omega'} \to \mathbb{R}, \ \lambda(\omega^{(0)}, \omega^{(1)}, \ldots, \omega^{(n)}) = \prod_{i=1}^{n} \beta^{\{(\omega^{(0)}, \omega^{(1)}, \ldots, \omega^{(i-1)})\}}_{\omega^{(i-1)}\omega^{(i)}}$$

is the share of ω' in ω's input mix through a given product chain. Hence, $\delta_{\omega\omega'}$ is the average number of production steps between an output ω and an input ω', weighted by the overall expenditure of each product chain in industry ω's mix of materials, and excluding any circular product chains.

The plant-level measure of the vertical span of production, and the left-hand side of Table III, is then the average distance of its inputs from the output, weighted by each inputs' share in the plants materials expenditure: let $j \in J_\omega$, then

$$\text{verticalSpan}_j = \sum_{\omega' \in \Omega} \frac{X_{j\omega'}}{\sum_{\omega'' \in \Omega} X_{j\omega''}} \delta_{\omega\omega'}.$$

To understand why we exclude circular product chains, consider the following example: Some plants sell aluminum and use aluminum scrap as an input, whereas some other plants use aluminum as an input and sell aluminum scrap. Thus in the production of aluminum scrap, aluminum would show up as an input one stage away, three stages away, five stages away, etc.

When we see a plant selling aluminum scrap and using aluminum as an input, we believe that it is the distance of one that is relevant, not the distances of three, five, etc. In other words, we believe the plant is turning the aluminum into scrap, but not turning the aluminum into scrap then back to aluminum and then back to scrap. Therefore we believe the circular part of the production chain is not relevant for constructing a plant's distance to its inputs.

B.2 Examples

Table B.1 shows the average vertical distance of several input groups (defined as all inputs that contain the strings "fabric"/"cloth", "yarn", or "cotton, raw" in their description") from the output "cotton shirts". Fabrics and cloths are closest to the final output; yarns, which are used in the production of cloths and fabrics, are further upstream, and raw cotton inputs are even further upstream.

Table B.2 shows vertical distances between aluminium ingots as an output, and several intermediate inputs. Aluminium ingots can be made both by recycling castings and alloys, but also by casting molten aluminium. The latter also serves as an intermediate input in the production of castings and alloys, and is hence vertically more distant than the inputs which undergo recycling. Aluminium itself is produced from aluminium oxide using electrolytic reduction (Hall-Heroult process). In turn, aluminium oxide is produced by dissolving bauxite in caustic soda at high temperatures (hence the coal inputs).

Table B.1 Vertical distance examples for 63428: Cotton Shirts

Input group	Average vertical distance
Fabrics Or Cloths	1.67
Yarns	2.78
Raw Cotton	3.55

Table B.2 Vertical distance examples for 73107: Aluminium Ingots

ASIC code	Input description	Vertical distance
73105	Aluminium Casting	1.23
73104	Aluminium Alloys	1.46
73103	Aluminium	1.92
22301	Alumina (Aluminium Oxide)	2.92
31301	Caustic Soda (Sodium Hydroxide)	3.81
23107	Coal	3.85
22304	Bauxite, raw	3.93

C Additional Reduced Form Results

C.1 Controlling for Interactions with State and Industry Characteristics

Tables C.1, C.2, and C.3 show the main regressions of materials cost shares, input mixes, and vertical span with a full set of controls. See Appendix A for definitions of the variables.

Tables C.4 and C.6 also include interactions of industry characteristics with court quality. Tables C.5 and C.7 show IV regressions where the court quality × industry characteristic interactions are instrumented by the interaction of log court age and the corresponding industry characteristic.

Table C.1 Additional Controls – Materials Cost Share

	Dependent variable: Materials Expenditure in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0167**	-0.0118*	-0.0156^{+}	-0.0212**
	(0.0046)	(0.0053)	(0.0085)	(0.0078)
LogGDPC * Rel. Spec.		0.0102		0.00556
		(0.0091)		(0.0096)
Trust * Rel. Spec.		0.0300		0.0353
		(0.038)		(0.038)
Language HHI * Rel. Spec.		0.0610		0.0612
		(0.040)		(0.040)
Caste HHI * Rel. Spec.		0.106*		0.0990^{+}
		(0.051)		(0.052)
Corruption * Rel. Spec.		0.0641		0.0508
		(0.097)		(0.097)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.480	0.484	0.480	0.484
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table C.2 Additional Controls – Vertical Span

	Dependent variable: Vertical Span			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	0.0195^{+}	0.0280*	0.0292	0.0368*
	(0.011)	(0.012)	(0.019)	(0.018)
LogGDPC * Rel. Spec.		0.0288		0.0330
		(0.024)		(0.024)
Trust * Rel. Spec.		-0.0939		-0.0984
		(0.090)		(0.091)
Language HHI * Rel. Spec.		-0.0742		-0.0743
		(0.092)		(0.092)
Caste HHI * Rel. Spec.		-0.182		-0.176
		(0.12)		(0.12)
Corruption * Rel. Spec.		0.481*		0.494*
		(0.24)		(0.24)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.443	0.453	0.443	0.453
Observations	163334	154021	163334	154021

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table C.3 Additional Controls – Input Mix

	Dependent variable: $X_j^R/(X_j^R+X_j^H)$			
	(1)	(2)	(3)	(4)
Avg age of Civil HC cases	-0.00547*	-0.00530*	-0.0144**	-0.0167**
	(0.0022)	(0.0024)	(0.0044)	(0.0045)
Log district GDP/capita		-0.00384		-0.00980$^+$
		(0.0046)		(0.0051)
Trust		-0.00740		-0.00160
		(0.018)		(0.019)
Language HHI		-0.0553**		-0.0567**
		(0.021)		(0.022)
Caste HHI		-0.0428		-0.0525$^+$
		(0.028)		(0.029)
Corruption		-0.0676		-0.0844$^+$
		(0.044)		(0.045)
5-digit Industry FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.441	0.449	0.441	0.449
Observations	225590	199339	225590	199339

Standard errors in parentheses, clustered at the state × industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table C.4 Materials Cost Share: Industry Characteristic Interactions

	Dependent variable: Materials Expenditure in Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0117*	-0.00987*	-0.0118*	-0.0112*	-0.0105$^+$	-0.00675
	(0.0052)	(0.0049)	(0.0053)	(0.0053)	(0.0054)	(0.0049)
Capital Intensity * Avg. age of cases	-0.110**					-0.0624$^+$
	(0.038)					(0.033)
Ind. Wage Premium * Avg. age of cases		-0.00146				-0.00180*
		(0.0011)				(0.00088)
Ind. Contract Worker Share * Avg. age of cases			-0.00116			0.0207
			(0.030)			(0.027)
Upstreamness * Avg. age of cases				0.00289$^+$		0.00363*
				(0.0015)		(0.0015)
Tradability * Avg. age of cases					-0.00104$^+$	-0.00150**
					(0.00058)	(0.00051)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.484	0.484	0.484	0.484	0.484	0.484
Observations	196748	196748	196748	196748	196735	196735

Standard errors in parentheses, clustered at the state × industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table C.5 Materials Cost Share: Industry Characteristic Interactions (IV)

	Dependent variable: Materials Expenditure in Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0211**	-0.0155*	-0.0211**	-0.0194*	-0.0217**	-0.0149*
	(0.0078)	(0.0074)	(0.0077)	(0.0080)	(0.0078)	(0.0075)
Capital Intensity * Avg. age of cases	-0.0262					-0.00252
	(0.065)					(0.062)
Ind. Wage Premium * Avg. age of cases		-0.00406*				-0.00343*
		(0.0018)				(0.0016)
Ind. Contract Worker Share * Avg. age of cases			0.0311			0.00893
			(0.040)			(0.041)
Upstreamness * Avg. age of cases				0.00641*		0.00593^{+}
				(0.0032)		(0.0031)
Tradability * Avg. age of cases					0.00108	0.000551
					(0.00100)	(0.0010)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.484	0.484	0.484	0.484	0.483	0.483
Observations	196748	196748	196748	196748	196735	196735

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table C.6 Vertical Span: Industry Characteristic Interactions

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	0.0279^{*}	0.0232^{+}	0.0278^{*}	0.0275^{*}	0.0295^{*}	0.0247^{*}
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)
Capital Intensity * Avg. age of cases	0.0279					0.0255
	(0.073)					(0.074)
Ind. Wage Premium * Avg. age of cases		0.00357^{+}				0.00299
		(0.0021)				(0.0021)
Ind. Contract Worker Share * Avg. age of cases			-0.0263			0.00432
			(0.026)			(0.033)
Upstreamness * Avg. age of cases				-0.00447		-0.00363
				(0.0036)		(0.0036)
Tradability * Avg. age of cases					-0.00118^{*}	-0.000961
					(0.00059)	(0.00079)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.453	0.453	0.453	0.453	0.453	0.453
Observations	154021	154021	154021	154021	154011	154011

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table C.7 Vertical Span: Industry Characteristic Interactions (IV)

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	0.0351*	0.0211	0.0362*	0.0360*	0.0388*	0.0257
	(0.018)	(0.019)	(0.018)	(0.018)	(0.018)	(0.019)
Capital Intensity * Avg. age of cases	0.259^{+}					0.370*
	(0.15)					(0.17)
Ind. Wage Premium * Avg. age of cases		0.0110**				0.00927*
		(0.0042)				(0.0043)
Ind. Contract Worker Share * Avg. age of cases			-0.0483			0.110^{+}
			(0.047)			(0.060)
Upstreamness * Avg. age of cases				-0.00405		0.00258
				(0.0070)		(0.0070)
Tradability * Avg. age of cases					-0.00397**	-0.00544**
					(0.0013)	(0.0018)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.453	0.453	0.453	0.453	0.453	0.453
Observations	154021	154021	154021	154021	154011	154011

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

C.2 Single-plant vs. Multi-plant Units

While we cannot link plants to firms, we have information about whether each plant belongs to a firm that has other plants. Tables C.8, C.9, C.10 show the core results for plants that belong to single-plant firms (indicated by a dummy), compared to plants that have sister plants, or for which it is not known whether they do. The relationship between materials share and court quality is not different for single-plant firms (same for the relationship between the input mix and court quality). On the other hand, the relationship between vertical span and court quality is only present for single-plant firms. Perhaps multi-plant firms have other ways of adjusting their organization to contracting frictions.

Table C.8 Materials Cost Share: Single-plant vs. Multi-plant units

	Dependent variable: Materials Expenditure in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0159**	-0.0109*	-0.0144^{+}	-0.0201*
	(0.0048)	(0.0055)	(0.0086)	(0.0079)
LogGDPC * Rel. Spec.		0.0101		0.00513
		(0.0091)		(0.0096)
Avg Age Of Civil Cases * Rel. Spec. * Single-plant firm	-0.00103	-0.00119	-0.00267	-0.00221
	(0.0017)	(0.0018)	(0.0019)	(0.0019)
Single-plant firm	-0.0151**	-0.0150**	-0.0129**	-0.0136**
	(0.0025)	(0.0026)	(0.0026)	(0.0027)
Rel. Spec. × State Controls		Yes		Yes
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.481	0.485	0.481	0.485
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level. Single-product plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"Rel. Spec. × State Controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table C.9 Input Mix: Single-plant vs. Multi-plant units

	Dependent variable: $X_j^R/(X_j^R + X_j^H)$			
	(1)	(2)	(3)	(4)
Avg age of Civil HC cases	-0.00826**	-0.00809**	-0.0143**	-0.0150**
	(0.0025)	(0.0028)	(0.0051)	(0.0052)
Avg Age Of Civil HC Cases * Single-plant firm	0.00482	0.00445	-0.00122	-0.00292
	(0.0032)	(0.0034)	(0.0063)	(0.0068)
Single-plant firm	-0.0371**	-0.0343**	-0.0203	-0.0138
	(0.0093)	(0.010)	(0.017)	(0.019)
Log district GDP/capita		-0.00412		-0.0104*
		(0.0045)		(0.0051)
State Controls		Yes		Yes
5-digit Industry FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.442	0.450	0.441	0.449
Observations	225590	199339	225590	199339

Standard errors in parentheses, clustered at the state × industry level. Single-product plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State Controls" are trust, language herfindahl, caste herfindahl, and corruption.

Table C.10 Vertical Span: Single-plant vs. Multi-plant units

	Dependent variable: Vertical Span			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0101	-0.00297	-0.00300	-0.00150
	(0.012)	(0.013)	(0.020)	(0.019)
Avg Age Of Civil Cases * Rel. Spec. * Single-plant firm	0.0347**	0.0381**	0.0467**	0.0498**
	(0.0061)	(0.0062)	(0.0075)	(0.0078)
Single-plant firm	0.0555**	0.0488**	0.0400**	0.0334**
	(0.0085)	(0.0089)	(0.0094)	(0.0098)
LogGDPC * Rel. Spec.		0.0375		0.0448^{+}
		(0.023)		(0.024)
Rel. Spec. × State Controls		Yes		Yes
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.446	0.456	0.446	0.456
Observations	163334	154021	163334	154021

Standard errors in parentheses, clustered at the state × industry level. Single-product plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"Rel. Spec. × State controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

C.3 Time variation in court quality

The regressions in the main text use variation in the average age of pending cases in High Courts to identify the impact of court congestion on input use. The microdata that is underlying the construction of this measure is available for one point in time only, meaning the regressions are exploiting entirely cross-sectional variation. In this section we try to use two different time-varying measures of court quality.

C.3.1 Creation of new High Court benches

Our first set of results using time variation in court congestion exploits two episodes where High Courts created new benches in cities further away from the main bench, with the specific aim of increasing access to justice in these remote areas. Two new benches of the Karnataka High Court were set up Dharwad and Gulbarga in July 2008. They have jurisdiction over Belgaum, Balgakot, Koppal, Gadag, Dharwad, Uttara Kannada, Haveri, and Bellary (Dharwad bench), and Bijapur, Gulbarga, Bidar, and Raichur (Gulbarga bench). Similarly, in July 2004, the Chennai High Court set up a bench in Madurai, which has jurisdiction over Kanniyakumari, Tirunelveli, Tuticorin, Madurai, Dindigul, Ramanathapuram, Virudhunagar, Sivaganga, Pudukkottai, Thanjavur, Tiruchirappalli and Karur districts.[3]

The regressions in Table C.11 look at whether wedges on relationship-specific inputs have decreased differentially in districts that are under the jurisdiction of the new benches. The coefficients are not very precisely estimated, since the districts with new benches account for few (about 6% on average) of the plants in the respective states. Nevertheless, the results are qualitatively consistent with those from the main text.

Table C.11 Identification from Time Variation: Diff-in-Diff

	X^R/Sales	$s_R - s_H$	Materials/TotalCost	Vert. Distance
	(1)	(2)	(3)	(4)
(New Bench in District)$_d\times$ (Post)$_t$	0.0126**	0.00960	-0.00305	0.00678
	(0.0043)	(0.0076)	(0.0033)	(0.010)
(New Bench in District)$_d\times$ (Post)$_t\times$ (Rel.Spec)$_\omega$			0.0142	-0.0764*
			(0.010)	(0.031)
Plant $\times$ Product FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
R^2	0.832	0.824	0.906	0.813
Observations	80427	74696	78462	77995

Figures C.1 and C.2 show the relative changes in the input mix in treated vs. untreated districts before and after the new high court benches were installed.

C.3.2 Pendency Ratios

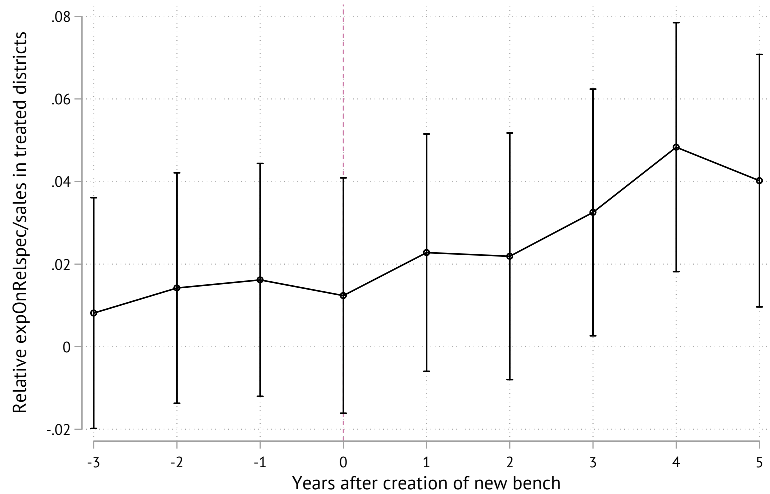
Our second set of regressions uses high court congestion ratios that vary by year, and that are published for a subset of High Courts in the 245th report of the Law Commission of India. For each year between 2002 and 2012, the report publishes the number of new and disposed cases during the year, and pending cases at the end of the year. We calculate the congestion ratio as

$$\text{Congestion Ratio}_{st} = \frac{(\text{Pending Cases})_{st}}{(\text{Disposed Cases})_{st}}.$$

This ratio can be interpreted as the number of years it takes to dispose of the backlog, if the number of disposed cases is constant over time.

[3]Source: Karnataka and Madras High Court websites: https://karnatakajudiciary.kar.nic.in/ and http://www.hcmadras.tn.nic.in/

Figure C.1 Relative change in X^R/Sales after new court bench is set up



The figure shows the evolution of the share of expenditure on relationship-specific inputs in sales, in treated districts relative to non-treated districts. Treatment happens at the start of period 0. Regression includes firm × product fixed effects and year dummies.

We should stress that these congestion ratios are not a good way to measure the cross-sectional variation in court speed. The Law Commission report collects these data from surveys of the High Courts, and mentions explicitly that different courts measure cases (as well as institution and disposal of cases) very differently, making comparisons problematic. However, if the cases and flows are measured in a consistent way over time, we may still use them for regressions where we compare input use and court quality over time, within state-industry pairs. Tables C.12 to C.14 show these regressions. While these results cannot be taken as evidence for a causal channel (we do not know what drives changes in pendency ratios), it is worth noting that they are consistent with the results from Section 2.

Table C.12 Materials Shares and Court Quality – Time variation

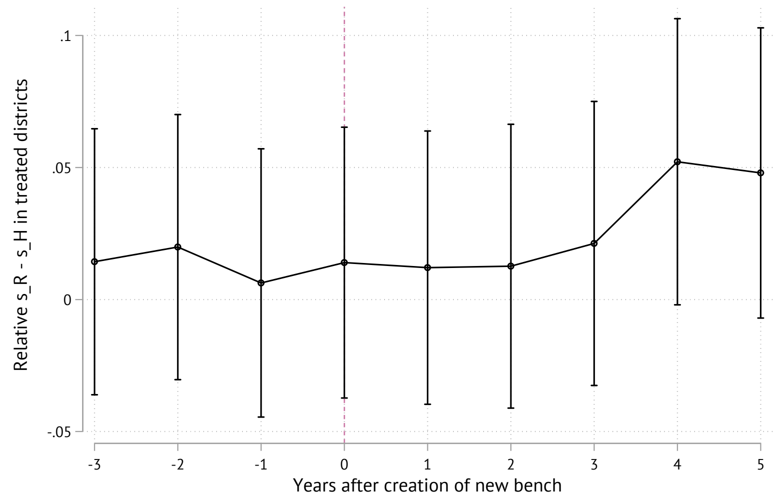
	Mat. Exp. / Total Cost
	(1)
Court Congestion Ratio * Rel. Spec.	-0.0573** (0.0066)
Rel. Spec. × State Controls	Yes
Industry × District FE	Yes
Estimator	OLS
R^2	0.718
Observations	86309

Standard errors in parentheses, clustered at the state × industry level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

"Rel. Spec. × State Controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Figure C.2 Relative change in composition of input mix after new court bench is set up



The figure shows the evolution of $s^R - s^H$, in treated districts relative to non-treated districts. Treatment happens at the start of period 0. Regression includes firm × product fixed effects and year dummies.

Table C.13 Input Mix and Court Quality – Time variation

	Dependent variable: $X_j^R/(X_j^R + X_j^H)$
	(1)
Court Congestion Ratio	-0.0118**
	(0.0041)
State Controls	Yes
Industry × District FE	Yes
Estimator	OLS
R^2	0.661
Observations	87936

Standard errors in parentheses, clustered at the state × industry level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

"State Controls" are trust, language herfindahl, caste herfindahl, and corruption.

Table C.14 Vertical Distance and Court Quality – Time variation

	Dependent variable: Vertical Span
	(1)
Court Congestion Ratio * Rel. Spec.	0.0164
	(0.010)
Rel. Spec. $\times$ State Controls	Yes
Industry $\times$ District FE	Yes
Estimator	OLS
R^2	0.660
Observations	71667

Standard errors in parentheses, clustered at the state $\times$ industry level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

"Rel. Spec. $\times$ State controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

C.4 Distortions and International Sourcing

This section presents three tables describing how plants' usage of domestically- and foreign-sourced inputs varies with distortions. The first and second columns of Table C.15 shows that, relative to those in industries that tend to use standardized inputs, the expenditure shares on domestic inputs of plants in industries that use relationship-specific inputs declines as courts get more congested. The third and fourth columns of Table C.15 shows that this estimated relationship is stronger when we use our instrumental variables strategy with court age as an instrument for congestion.

Table C.15 Domestic Materials Shares and Court Quality (Fact 3)

	Dependent variable: Dom. Mat. Exp. in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0110^{+}	-0.0127*	-0.0349**	-0.0279**
	(0.0057)	(0.0063)	(0.0097)	(0.0085)
LogGDPC * Rel. Spec.		-0.0124		-0.0200^{+}
		(0.0098)		(0.010)
Trust * Rel. Spec.		-0.0415		-0.0331
		(0.043)		(0.043)
Language HHI * Rel. Spec.		0.00109		0.00139
		(0.056)		(0.056)
Caste HHI * Rel. Spec.		0.0466		0.0355
		(0.074)		(0.075)
Corruption * Rel. Spec.		0.0654		0.0439
		(0.12)		(0.12)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.432	0.439	0.431	0.439
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table C.16 shows the same regressions but with the cost share of imported intermediates. Here, the OLS and IV specifications point to opposite results, and further investigation is required. The OLS specifications indicate that among those that rely more on more relationship-specific inputs, more congestion leads to lower shares of imported inputs relative to those that rely on standardized inputs. The IV specification indicates that more congestion leads to relatively higher imported input shares in those that rely on relationship-specific inputs, indicating that firms respond to distortions by substituting from domestic to foreign suppliers.

Table C.17 shows the results on the share of imports in the basket of relationship-specific (first two columns) and homogenous (last two columns) inputs. More congestion leads to a higher share of imports in both baskets, but the substitution is stronger in the basket of relationship-specific goods.

Table C.16 Import Shares and Court Quality, OLS + IV

	Dependent variable: Imported Materials Expenditure in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.00563***	0.000870	0.0193***	0.00666**
	(0.0013)	(0.0014)	(0.0025)	(0.0022)
LogGDPC * Rel. Spec.		0.0227***		0.0255***
		(0.0027)		(0.0028)
Trust * Rel. Spec.		0.0716***		0.0683***
		(0.011)		(0.011)
Language HHI * Rel. Spec.		0.0599***		0.0598***
		(0.012)		(0.012)
Caste HHI * Rel. Spec.		0.0593***		0.0635***
		(0.016)		(0.016)
Corruption * Rel. Spec.		-0.00127		0.00691
		(0.031)		(0.031)
Constant		-0.113***		
		(0.015)		
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.330	0.342	0.329	0.342
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table C.17 Substitution into Importing

	R-Imports in Total R		H-Imports in Total H	
	(1)	(2)	(3)	(4)
Avg age of Civil HC cases	0.0193**	0.00925**	0.0112**	0.00440**
	(0.0023)	(0.0018)	(0.0016)	(0.0013)
Log district GDP/capita		0.0224**		0.0180**
		(0.0027)		(0.0019)
Trust in other people (WVS)		0.110**		0.0564**
		(0.012)		(0.011)
Language Herfindahl		0.0162		-0.0292**
		(0.019)		(0.0093)
Caste Herfindahl		0.0584*		0.0171
		(0.028)		(0.013)
Corruption		0.0315		-0.0912**
		(0.028)		(0.022)
5-digit Industry FE	Yes	Yes	Yes	Yes
Estimator	IV	IV	IV	IV
R^2	0.227	0.251	0.180	0.197
Observations	168120	148165	168953	149623

Standard errors in parentheses, clustered at the state × industry level.
Dependent variable in columns (1) and (2) (resp. (3) and (4)) is the share of relationship-specific (homogeneous) imports in total relationship-specific (homogeneous) materials.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

C.5 Materials Shares with Size and Age

Table C.18 shows that materials cost shares do not correlate much with size and age of the plant. Table C.19 shows correlations of input wedges with various plant-level characteristics.

Table C.18 Plant Age and Size

	Dependent variable: Mat. Exp in Total Cost		
	(1)	(2)	(3)
Plant Age	-0.000695**		-0.000679**
	(0.000065)		(0.000063)
Log Employment		-0.00257**	-0.00176*
		(0.00086)	(0.00083)
5-digit Industry FE	Yes	Yes	Yes
District FE	Yes	Yes	Yes
Estimator			
R^2	0.481	0.481	0.482
Observations	205109	208179	204767

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table C.19 Wedges and Plant Characteristics

	Age	Size	Multiproduct	# Products
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	0.620+	-0.0253	-0.0121	-0.0580
	(0.32)	(0.040)	(0.0076)	(0.037)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
R^2	0.214	0.339	0.301	0.295
Observations	353392	359820	360316	360316

Note: Sample includes multiproduct plants. Industry dummies refer to the 5-digit industry with the plants' highest production value.

D Exploring the Nature of Contracting Frictions

What do these regressions tell us about the form of contracting frictions? The literature on contracting frictions (e.g., Antràs (2003)) has emphasized that holdup problems may result in transactions in which the seller shades on the quantity or quality of the inputs. These can be modeled in different ways that would show up differently in the data.

If distortions resulted in an inefficiently low quantity of the input, the buyer's shadow value of the input would be above the seller's marginal cost. In this formulation, the distortion would be observationally equivalent to a positive wedge as in Hsieh and Klenow (2009). One testable implication of this formulation is that the distortion should raise the buyer's ratio of revenue to total cost. In Appendix D.1, we explore this relationship in our setting. We find that plants that are subject to larger wedges—those in industries that tend to use relationship-specific inputs in states with congested courts—have *lower* revenue-cost ratios, in contrast to the prediction from a quantity distortion.[4]

[4] The negative correlation of price-cost margins with distortions holds even when controlling for plant size and age.

Distortions that result in an inefficiently low quality of the input can take several forms, depending on the relationship between quality and quantity. The simplest form is that quality is quantity-augmenting, in which case the distortion would be equivalent to a higher effective price of the input (i.e. an iceberg cost). In this case, the impact on cost shares depends on the elasticity of substitution between distorted and undistorted inputs. If primary inputs and intermediate inputs were substitutes, a higher effective price of the input would cause the cost share of intermediates to fall. However, we believe the evidence does not support an elasticity of substitution between primary and intermediate inputs that is larger than unity. We know of two estimates of long-run plant level elasticities of substitution between materials and primary inputs, Oberfield and Raval (2014) and Appendix B.3 of Atalay (2017). Each find elasticities that are slightly less than unity. Further, we can investigate this elasticity in the context of Indian manufacturing plants. In Appendix D.2 we use upstream contracting distortions as a shifter of the seller's costs. We find evidence against an elasticity greater than one, in line with what the existing literature finds.

Finally, it could be that quality and quantity enter the production function in different ways. For example, it may be that the seller needs to customize the good for the buyer and can do so inexpensively, and the buyer can do the customization herself but less efficiently than the seller. In that case, if the friction causes the seller to insufficiently customize the good, the buyer's cost will rise because the wrong producer is doing the customization. Further, part of this effective expenditure on the distorted input will appear in the data as an expenditure by the buyer on primary factors–the primary factors used by the buyer to finish the customization of the good. Thus the expenditure share on intermediates (and especially on relationship-specific intermediates) will be lower when distortions are more prevalent, even if the elasticity of substitution between primary and intermediate inputs is weakly less than one. This last remaining way of modeling distortions is consistent with the regressions of materials cost shares (Table I) and the mix of distorted vs undistorted materials inputs (Table II). We will therefore use it in the subsequent model.[5]

D.1 Distortions and Revenue-Cost Margins

Table D.1 shows how plants' ratios of sales to cost vary with the level of distortions. Court congestion should increase distortions for plants in industries that tend to rely on relationship-specific inputs relative to those in industries that tend to rely on homogeneous inputs. The first row of the table indicates that, across all measures, ratios of sales to cost decline with distortions. The second and third column shows that this appears to be the direct impact of the distortions rather than the indirect impact of distortions on size or age.

[5]It could be that the buyer and seller are equally good at customization, in which case the friction simply leads to a different division of labor without raising the buyers marginal cost. However, we find that distortions reduce entry (see Appendix D.3), suggesting that they do increase marginal cost.

Table D.1 Sales over Total Cost

	Dependent variable: Sales/Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0353**	-0.0347**	-0.0345**	-0.0494*	-0.0496*	-0.0508*
	(0.0097)	(0.0094)	(0.0093)	(0.022)	(0.022)	(0.022)
Plant Age		0.000574**	0.000258^{+}		0.000575**	0.000259^{+}
		(0.00014)	(0.00014)		(0.00014)	(0.00014)
Log Employment			0.0314**			0.0314**
			(0.0016)			(0.0016)
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.114	0.110	0.115	0.114	0.110	0.115
Observations	208527	205109	204767	208527	205109	204767

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

D.2 Plant-level substitutability of inputs

Fact 1 showed that when relationship-specific inputs are more severely distorted, the cost share of those inputs declines. We posit in Section 2.5 that this happens because the buyer must expend additional primary inputs in order to use the relationship-specific input. An alternative possibility is that the distortion raises the price paid to the supplier. If primary inputs and intermediate inputs were substitutes, the cost share of intermediates would fall. However, we believe the evidence does not support an elasticity of substitution between primary and intermediate inputs that is larger than unity.

We know of two estimates of long-run plant level elasticities of substitution between materials and primary inputs, Oberfield and Raval (2014) and Appendix B.3 of Atalay (2017). Each find elasticities slightly lower than unity. Further, we can investigate this elasticity in our context. Table D.2 uses the interaction of court quality with the average dependence on relationship-specific inputs among an industry's upstream industries to stand in for a shifter to the cost of intermediate inputs. The coefficient of interest is in the third row. Neither the OLS regressions in columns (1) and (2) nor the IV regressions in columns (3) and (4) provide support for an elasticity of substitution larger than one, which would require a negative coefficient. Thus, we do not find support for an assertion that primary and intermediate inputs are substitutes at the plant-level.

Table D.2 Plant-level elasticity of substitution

	Dependent variable: Materials Expenditure in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0147^{+}	-0.0135	-0.0397**	-0.0401**
	(0.0080)	(0.0089)	(0.013)	(0.013)
LogGDPC * Rel. Spec.		0.0112		0.00653
		(0.0090)		(0.0095)
Avg Age Of Civ. Cases * Rel. Spec. of Upstream Sector	-0.00360	0.00268	0.0450*	0.0349^{+}
	(0.011)	(0.012)	(0.019)	(0.020)
Trust * Rel. Spec.		0.0274		0.0342
		(0.038)		(0.038)
Language HHI * Rel. Spec.		0.0467		0.0501
		(0.032)		(0.032)
Caste HHI * Rel. Spec.		0.0980*		0.0897^{+}
		(0.050)		(0.050)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.480	0.484	0.480	0.484
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

D.3 Distortions are costly

In our microfoundation, the reason a distortion is costly is that the resource the supplier saves by delivering an imperfect input is smaller than the resources used by the buyer correcting the input. An alternative possibility is that these costs are roughly equal, in which case there would be no resource cost of the distortion despite the fact that the buyer's reduced cost share of relationship-specific inputs. In the extreme, this would mean that despite the impact on cost shares, the loss in productivity would be smaller than suggested by Section 4.2. We argue here that distortions do indeed raise the buyer's cost. Table D.3 shows that when distortions are more likely to be severe—in industries that tend to rely on relationship-specific inputs in states with slower courts—industries tend to have fewer plants.

Table D.3 Extensive-margin regressions

	Dependent variable: $\log \lvert J^d_{\omega,t} \rvert$					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0413*	-0.0382*	-0.0259	-0.133**	-0.107**	-0.106**
	(0.017)	(0.017)	(0.018)	(0.035)	(0.035)	(0.032)
LogGDPC * Rel. Spec.		0.0505**	0.0518**		0.0313	0.0353*
		(0.016)	(0.016)		(0.019)	(0.018)
Trust * Rel. Spec.			-0.0482			0.00793
			(0.14)			(0.14)
Language HHI * Rel. Spec.			0.120			0.121
			(0.15)			(0.15)
Caste HHI * Rel. Spec.			0.225**			0.171^{+}
			(0.086)			(0.087)
Corruption * Rel. Spec.			0.537^{+}			0.381
			(0.32)			(0.33)
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
State × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.410	0.417	0.423	0.410	0.417	0.423
Observations	191008	183214	177075	191008	183214	177075

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Note: Dependent variable is the log number of producers of ω in a state d at time t. Multi-product plants are counted once for each product. GDP per capita is the average district GDP per capita within each state.

D.4 Do distortions cause the buyer to use one particular component of primary inputs?

In Section 2.5 we posit that when a relationship-specific input is distorted, the buyer must use primary inputs to complete the customization. Primary inputs includes labor, capital, services, and some other inputs. We explore now whether the the distortion affects spending on any particular primary input more than others. Table D.4 shows regressions of the cost share of labor/capital/services/other inputs in total non-material inputs on the interaction of court quality and relationship-specificity. If one particular component of primary inputs was used to customize the distorted input, or if the distorted intermediate input were particularly complementary with one particular component, then we should find that the share of that input in non-materials costs should be higher whenever the contracting frictions are stronger. We find that this is not the case, suggesting that distortions are being paid in the overall bundle of primary inputs.

Table D.4 Cost share of factors in primary inputs

	Labor	Capital	Services	Rest
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.000504	0.00405	-0.000248	-0.00355
	(0.0054)	(0.0030)	(0.0046)	(0.0049)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
R^2	0.310	0.232	0.341	0.269
Observations	208527	208527	208527	208527

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
The dependent variable is the cost share of the factor in the total cost of non-material inputs.

E Imperfect Contract Enforcement

Suppliers can sell a good that is defective or imperfectly customized to the buyer. If this happens, the buyer must use labor to correct the defect or complete the customization. In principle, the supplier can save on production cost by producing defective/imperfectly customized inputs. To produce one unit of the intermediate input that is defective enough so that the buyer must use up ψ^x units of labor, the supplier's unit cost is $c_s(\psi)$, where $c_s(0) \equiv c_s$ is the supplier's unit cost of producing a defect-free unit of the input. We assume that $c_s'(\psi) \geq -w$, which ensures that the sum of the buyer's and supplier's payoffs are maximized at $\psi = 0$.

A contract between buyer and supplier is a triple (M^c, ψ^c, x^c), where $x^c \geq 0$ is the quantity of the good to be delivered, ψ^c is a desired customization level, M^c is a payment from the buyer to the supplier upon delivery. We assume that quantity x is costlessly enforceable, which ensures that the supplier chooses $x = x^c$.

Both the buyer and supplier anticipate equilibrium behavior. In line with the main text, we assume that the buyer has full bargaining power, in that she can make a take-it-or-leave-it offer to the supplier.

If the contract has been breached (either because the supplier chooses a $\psi > \psi^c$ or because the buyer chooses $M < M^c$), either party could enforce the contract in a court. The outcome of enforcement is deterministic, and enforcement is costly. The plaintiff has to pay enforcement costs, which amount to a proportion π of the value of the transaction (M^c), while the defendant expends δ of the value of the transaction. The value of the claim to the plaintiff is the net transfer to her that would arise under enforcement. Enforcement costs cannot be recovered in court. The enforcement cost for the plaintiff $\pi \in (0, 1)$ is randomly drawn for each buyer-supplier pair.

In principle, the contracts can be written to get around this friction. However, the doctrine of *expectation damages* limits the damages the buyer can collect from the supplier. The damages cannot be more than what is needed so that the buyer's is as well off as she would have been had the contract been honored.[6]

Thus if the supplier breaches by choosing $\psi > \psi^c$ and the buyer enforces the contract in court, the buyer receives a gross transfer of $w\psi x - w\psi^c x$. Thus net of enforcement costs, the buyer receives $w\psi x - w\psi^c x - \pi M^c$, and the supplier pays $w\psi x - w\psi^c x + \delta M^c$. If the buyer chooses $M < M^c$, the court orders the buyer to pay $M^c - M$ to the buyer. Thus net of enforcement costs, the supplier receives $M^c - M - \pi M^c$, and the buyer pays $M^c - M + \delta M^c$.

Given the choice of supplier, the buyer's payoff from the contract (up to an additive normalizing constant) is $-w\psi x - M$ plus the value of any net transfers mandated by the court. Similarly, the payoff to the supplier (up to an additive normalizing constant) can be expressed as $-c(\psi)x + M$ plus the value of any net transfers mandated by the court.

E.1 Timing of events

1. The buyer makes a take-it-or-leave-it offer of a contract, (M^c, ψ^c, x^c)

[6] See Shavell (1980) and Boehm (2018).

2. The supplier decides whether to accept the offer. If the supplier accepts the contract, proceed to the next step.
3. The supplier produces x units at defectiveness level $\psi \geq 0$ for a unit cost of $c_s(\psi)$.
4. The buyer makes a transfer M.
5. If the contract has been breached, either party could enforce the contract in a court.
6. Production occurs.

E.2 Solving the game

Lemma E.1 *$M^c > 0$, $M = (1-\pi) M^c$, $\psi = \psi^c + \pi \frac{M^c}{xw}$, and neither party sues for breach of contract.*

Proof. First, M^c must be strictly positive because otherwise the supplier would not agree to the contract. The supplier would sue if $(M^c - M) - \pi M^c > 0$. Thus it must be that $M \leq (1-\pi) M^c$ because otherwise the buyer could strictly improve her payoff by reducing M without risk of getting sued. On the other hand, if $M < (1-\pi) M^c$, the buyer could strictly increase her payoff by setting $M = (1-\pi) M^c + \epsilon$, for small enough ϵ, in which case the buyer would avoid getting sued and would avoid legal costs. Therefore it must be that $M \geq (1-\pi) M^c$. Together, these imply that $M = (1-\pi) M^c$. There cannot be an equilibrium in which $M = (1-\pi) M^c$ and the supplier sues with positive probability, because in that case, the buyer would have been strictly better off by setting $M = (1-\pi) M^c + \epsilon$, for small enough ϵ, and avoiding the legal costs of getting sued.

Similarly, the buyer would sue only if $\psi > \psi^c$ and $(\psi xw - \psi^c xw) - \pi M^c > 0$. It must be that $(\psi xw - \psi^c xw) - \pi M^c \geq 0$, because otherwise the supplier could strictly improve her payoff by raising ψ without risk of getting sued. On the other hand, if $\psi xw > \psi^c xw + \pi M^c$, the buyer would sue. Thus the supplier would be better off setting $\tilde{\psi} = \psi^c + \frac{\pi M^c}{xw} - \epsilon$ for small enough ϵ and avoiding the legal costs of getting sued: $\frac{c'}{w} > -1$ implies that the change in payoff is positive for small enough ϵ:

$$
\begin{aligned}
-c\left(\tilde{\psi}\right) x - \left[-c(\psi) x - (\psi xw - \psi^c xw) - \delta M^c\right] &= xw \int_{\tilde{\psi}}^{\psi} \frac{c'(u)}{w} du + (\psi xw - \psi^c xw) + \delta M^c \\
&> xw \int_{\tilde{\psi}}^{\psi} (-1)\, du + (\psi xw - \psi^c xw) + \delta M^c \\
&= xw \left(\tilde{\psi} - \psi\right) + (\psi xw - \psi^c xw) + \delta M^c \\
&= xw\tilde{\psi} - \psi^c xw + \delta M^c \\
&= xw \left(\psi^c + \frac{\pi M^c}{xw} - \epsilon\right) - \psi^c xw + \delta M^c \\
&= (\pi + \delta) M^c - \epsilon xw
\end{aligned}
$$

Therefore it must be that $\psi xw = \psi^c xw + \pi M^c$. Finally, there cannot be an equilibrium in which $\psi xw = \psi^c xw + \pi M^c$ and the buyer sues with positive probability because in that case the supplier would have been strictly better off by setting $\tilde{\psi} = \psi^c + \frac{\pi M^c}{xw} - \epsilon$. ■

Given those, we can find the payoff to the buyer and supplier of an arbitrary contract (M^c, ψ^c, x^c):

$$
\begin{aligned}
\text{buyer} &: \quad -\psi xw - M = -\psi^c xw - \pi M^c - (1-\pi) M^c \\
\text{supplier} &: \quad -c(\psi) x + M = -c\left(\psi^c + \frac{\pi M^c}{xw}\right) x + (1-\pi) M^c
\end{aligned}
$$

It will be convenient to define $p^c \equiv \frac{M^c}{x^c}$ and $p \equiv \frac{M}{x}$ to be the average price as specified in the contract and in equilibrium. With this, we can express the contract and payoffs in per unit terms:

$$\begin{aligned}\text{buyer} \quad &: \quad -\psi^c w - \pi p^c - (1-\pi)p^c \\ \text{supplier} \quad &: \quad -c\left(\psi^c + \frac{\pi p^c}{w}\right) + (1-\pi)p^c\end{aligned}$$

The buyer makes a take-it-or-leave-it offer of

$$\max_{p^c,\psi^c} -p^c - \psi^c w$$

subject to

$$-c\left(\psi^c + \frac{\pi p^c}{w}\right) + (1-\pi)p^c \geq 0$$

Lemma E.2 *Any contract for which the constraint does not bind can be improved upon.*

Proof. Suppose that (p^c, ψ^c) is a contract for which the constraint is not binding. Then consider the alternative contract $\left(\tilde{p}^c, \tilde{\psi}^c\right)$ in which $\tilde{p}^c = p^c - \epsilon$ and $\tilde{\psi}^c = \psi^c + \frac{\pi}{w}\epsilon$. In this alternative contract $\tilde{\psi}^c + \frac{\pi \tilde{p}^c}{w} = \psi^c + \frac{\pi p^c}{w}$. If ϵ is small enough, the constraint will not be violated, and the buyers payoff strictly rises:

$$-\tilde{p}^c - \tilde{\psi}^c w = -\left(p^c - \epsilon\right) - \left(\psi^c + \frac{\pi}{w}\epsilon\right) w = -p^c - \psi^c w + (1-\pi)\epsilon > -p^c - \psi^c w$$

■

Claim E.1 $\psi^c = 0$ *and* p^c *is the unique solution to* $(1-\pi)p^c = c\left(\frac{\pi p^c}{w}\right)$.

Proof. First, note that $c'(\psi) > -w$ implies that that if $\psi > \tilde{\psi}$, $-c\left(\tilde{\psi}\right) > -c(\psi) + w\left(\tilde{\psi} - \psi\right)$. Consider a contract (p^c, ψ^c) with $\psi^c > 0$. The buyer's problem can be expressed as

$$\max_{p^c,\psi^c} -p^c - \psi^c w \;\; \text{subject to} \;\; (1-\pi)p^c - c\left(\psi^c + \frac{\pi p^c}{w}\right) \geq 0.$$

Consider the alternative contract in which $\tilde{\psi}^c = \psi^c - \epsilon$ and $\tilde{p}^c = p^c + \epsilon w$. That alternative contract would leave the buyer with the same payoff. We next show that the constraint would not bind, which will imply that the contract can be improved upon. The left hand side of the constraint is

$$\begin{aligned} LHS &= (1-\pi)\tilde{p}^c - c\left(\tilde{\psi}^c + \frac{\pi \tilde{p}^c}{w}\right) \\ &> (1-\pi)\,\tilde{p}^c + \left\{-c\left(\psi^c + \frac{\pi p^c}{w}\right) + \left[\left(\tilde{\psi}^c + \frac{\pi \tilde{p}^c}{w}\right) - \left(\psi^c + \frac{\pi p^c}{w}\right)\right] w\right\} \\ &\geq (1-\pi)\tilde{p}^c + \left\{-(1-\pi)p^c + \left[\left(\tilde{\psi}^c + \frac{\pi \tilde{p}^c}{w}\right) - \left(\psi^c + \frac{\pi p^c}{w}\right)\right] w\right\} \\ &= \tilde{p}^c - p^c + \left(\tilde{\psi}^c - \psi^c\right) w \\ &= 0 \end{aligned}$$

where the weak inequality imposes the fact that the constraint holds for the original contract.

Imposing that $\phi^c = 0$, the buyer's problem can then be expressed as

$$\max_{p^c} -p^c \;\; \text{subject to} \;\; (1-\pi)p^c - c\left(\frac{\pi p^c}{w}\right) \geq 0$$

where the constraint binds. This means that p^c solves $(1-\pi)p^c = c\left(\frac{\pi p^c}{w}\right)$. Since the left hand side is increasing in p^c while the right hand side is decreasing in p^c, and since the two curves must cross at least once, there is a unique solution. ■

We study a limiting economy in which $c_s(\psi) \to c_s$, i.e., the supplier can customize the good at essentially no cost. For example, we could have $c(\psi) = \bar{c}e^{-\frac{bw\psi}{\bar{c}}}$, with $b < 1$, and study the limit as $b \to 0$. In this limit,

$$\begin{aligned} p^c &= \frac{\bar{c}}{1-\pi} \\ \psi^c &= 0 \\ p &= \bar{c} \\ \psi &= \frac{\pi p^c}{w} = \frac{\pi}{1-\pi}\frac{\bar{c}}{w} \end{aligned}$$

In this economy, π is drawn randomly. We define $t_x \equiv \frac{1}{1-\pi}$.

E.3 Production

Given prices and the defectiveness of each input, the buyer minimizes cost. Let l^{prod} be the mass of labor hired for production and let $l^x_{\hat{\omega}}$ be the mass of labor hired to customize the defective inputs. The firm's cost minimization problem can be described as:

$$\min w\left(l^{prod} + \sum_{\hat{\omega}\in\hat{\Omega}^\rho_R} l^x_{\hat{\omega}}\right) + \sum_{\hat{\omega}\in\Omega^\rho_R} p_{\hat{\omega}s}\left(\psi_{\hat{\omega}s}\right)x_{\hat{\omega}} + \sum_{\hat{\omega}\in\Omega^\rho_R} p_{\hat{\omega}s}x_{\hat{\omega}}$$

subject to

$$G\left(b_l l^{prod}, \left\{b_{\hat{\omega}}z_{\hat{\omega}}\min\left\{x_{\hat{\omega}}, \frac{l^x_{\hat{\omega}}}{\psi_{\hat{\omega}s}}\right\}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_R}, \left\{b_{\hat{\omega}}z_{\hat{\omega}s}x_{\hat{\omega}}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_H}\right) \geq y$$

This minimization problem can be rewritten as

$$\min wl^{prod} + \sum_{\hat{\omega}\in\Omega^\rho_R}\left(w\psi^x_{\hat{\omega}s} + p_{\hat{\omega}s}(\psi_{\hat{\omega}s})\right)x_{\hat{\omega}} + \sum_{\hat{\omega}\in\Omega^\rho_H} p_{\hat{\omega}s}x_{\hat{\omega}}$$

subject to

$$G\left(b_l l^{prod}, \left\{b_{\hat{\omega}}z_{\hat{\omega}s}x_{\hat{\omega}}\right\}_{\hat{\omega}\in\Omega^\rho}\right) \geq y$$

Thus if $\mathcal{C}$ is the unit cost function associated with G, the firms unit cost can be expressed as

$$\mathcal{C}\left(\frac{w}{b_l}, \left\{\frac{p_{\hat{\omega}s} + w\psi_{\hat{\omega}s}}{b_{\hat{\omega}}z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_R}, \left\{\frac{p_{\hat{\omega}s}}{b_{\hat{\omega}}z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_H}\right)$$

or, given the equilibrium actions of the buyers and suppliers,

$$\mathcal{C}\left(\frac{w}{b_l}, \left\{\frac{t_{\hat{\omega}s}p_{\hat{\omega}s}}{b_{\hat{\omega}}z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_R}, \left\{\frac{p_{\hat{\omega}s}}{b_{\hat{\omega}}z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_H}\right).$$

F Proofs

F.1 Proof of Proposition 1

Let $F_\omega(c)$ be the fraction of firms in industry ω with cost weakly less than c (when the wage is normalized to unity). Similarly, let $F_{\omega\rho}(c)$ be the fraction of firms in ω that have a technique of recipe ρ that delivers unit cost weakly less than c. These satisfy $1 - F_\omega(c) = \prod_{\rho\in\varrho(\omega)}\left[1 - F_{\omega\rho}(c)\right]$.

It will also be convenient to define, for recipe ρ that uses inputs $\hat{\Omega}^{\rho} = (\hat{\omega}_1, ..., \hat{\omega}_n)$, the functions $\mathcal{B}_{\omega\rho}(b)$ where $b = (b_l, b_1, ..., b_n)$ and $\mathcal{V}_{\omega\rho}(v)$ where $v = (v_l, v_1, ..., v_n)$. These functions are defined by $\mathcal{B}_{\omega\rho}(b) \equiv B_{\omega\rho} b_l^{-\beta_l^{\rho}} \prod_k b_k^{-\beta^{\rho}_{\hat{\omega}_k}}$ so that $\mathcal{B}_{\omega\rho}(db) = B_{\omega\rho}\beta_l^{\rho} b_l^{-\beta_l^{\rho}-1} db_l \beta^{\rho}_{\hat{\omega}_1} b_1^{-\beta^{\rho}_{\hat{\omega}_1}-1} db_1 ... \beta^{\rho}_{\hat{\omega}_n} b_n^{-\beta^{\rho}_{\hat{\omega}_n}-1} db_n$. Similarly, let $\mathcal{V}_{\omega\rho}(v) \equiv v_l^{\beta_l^{\rho}} \prod_k v_k^{\beta^{\rho}_{\hat{\omega}_k}}$ so that $\mathcal{V}_{\omega\rho}(dv) = \beta_l^{\rho} v_l^{\beta_l^{\rho}-1} dv_l \beta^{\rho}_{\hat{\omega}_1} v_1^{\beta^{\rho}_{\hat{\omega}_1}-1} dv_1 ... \beta^{\rho}_{\hat{\omega}_n} v_n^{\beta^{\rho}_{\hat{\omega}_n}-1} dv_n$.

Lemma F.1 *Under Assumption 2, for a firm of type ω,*

$$\Pr(\lambda_{\hat{\omega}}(\phi) > \lambda | b_{\hat{\omega}}(\phi)) = e^{-(\lambda b_{\hat{\omega}}(\phi)/\Lambda_{\hat{\omega}})^{\zeta_{\hat{\omega}}}}$$

where

$$\Lambda_{\hat{\omega}} = \begin{cases} t_x^* \left[\int_0^{\infty} c^{-\zeta_R} dF_{\hat{\omega}}(c)\right]^{-1/\zeta_R}, & \hat{\omega} \in \Omega_R^{\rho} \\ \left[\int_0^{\infty} c^{-\zeta_H} dF_{\hat{\omega}}(c)\right]^{-1/\zeta_H}, & \hat{\omega} \in \Omega_H^{\rho} \end{cases}$$

and

$$t_x^* \equiv \left(\int_1^{\infty} t_x^{-\zeta_R} dT(t_x)\right)^{-1/\zeta_R}$$

.

Proof. Consider first a relationship-specific input $\hat{\omega} \in \hat{\Omega}_R^{\rho}$. Consider a technique with a common component of input-augmenting productivity $b_{\hat{\omega}}(\phi)$. The number of suppliers with match-specific component of input-augmenting productivity greater than z is Poisson with mean $z^{-\zeta_R}$. For a potential supplier with z and input wedge t_x, the probability that the supplier's cost is low enough so that the supplier delivers an effective cost weakly less than λ is $\Pr\left(\frac{p_s t_x}{b_{\hat{\omega}}(\phi) z} \leq \lambda\right) = \Pr\left(p_s \leq \lambda z b_{\hat{\omega}}(\phi)/t_x\right) = F_{\hat{\omega}}\left(\lambda z b_{\hat{\omega}}(\phi)/t_x\right)$, where $F_{\hat{\omega}}(c)$ is the fraction of firms in $\hat{\omega}$ with unit cost (and hence price) weakly less than c. Integrating over realizations of z and t_x, we have that the number of potential suppliers that deliver effective cost weakly less than λ follows a Poisson distribution with mean

$$\int_0^{\infty} \int_1^{\infty} F_{\hat{\omega}}\left(\lambda z b_{\hat{\omega}}(\phi)/t_x\right) dT(t_x) \zeta_R z^{-\zeta_R - 1} dz$$

Using the change of variables $c = \lambda z b_{\hat{\omega}}(\phi)/t_x$ and the definition of $\Lambda_{\hat{\omega}}$, this is

$$[\lambda b_{\hat{\omega}}(\phi)]^{\zeta_R} \int_0^{\infty} \int_1^{\infty} F_{\hat{\omega}}(c) t_x^{-\zeta_R} dT(t_x) \zeta_R c^{-\zeta_R - 1} dv = [\lambda b_{\hat{\omega}}(\phi)/\Lambda_{\hat{\omega}}]^{\zeta_R}$$

The probability that no such suppliers arrive is then simply

$$\Pr(\lambda_{\hat{\omega}}(\phi) > \lambda | b_{\hat{\omega}}(\phi)) = e^{-(\lambda b_{\hat{\omega}}(\phi)/\Lambda_{\hat{\omega}})^{\zeta_R}}$$

The logic for homogeneous inputs is the same. ■

Lemma F.2 *Under Assumption 1,*

$$\int_0^{\infty} ... \int_0^{\infty} 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} \mathcal{V}_{\omega\rho}(dv) < \infty$$

Proof. Assumption 1 implies that for each $k \in \{l, 1, ..., n\}$ there is a $\bar{v}_k$ such that $\mathcal{C}_{\omega\rho}\left(0, ..., 0, \bar{v}_k, 0, ..., 0\right) = 1$. In other words, $\bar{v}_k$ is defined so that if the effective cost of the kth input were equal to $\bar{v}_k$ and the cost of all other inputs were equal to zero then the firm's cost would be 1. Thus if the firm's cost of the kth input were higher than $\bar{v}_k$, the firm's cost must be greater than 1. We therefore have

$$
\begin{aligned}
\int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} \mathcal{V}_{\omega\rho}(dv) &= \int_0^{\bar{v}_l} \int_0^{\bar{v}_1} ... \int_0^{\bar{v}_n} 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} \mathcal{V}_{\omega\rho}(dv) \\
&\leq \int_0^{\bar{v}_l} \int_0^{\bar{v}_1} ... \int_0^{\bar{v}_n} \mathcal{V}_{\omega\rho}(dv) \\
&= \mathcal{V}_{\omega\rho}(\bar{v}) \\
&< \infty
\end{aligned}
$$

where $\bar{v} = \{\bar{v}_l, \bar{v}_1, ..., \bar{v}_n\}$. ■

Proposition F.1 *Under Assumptions 1 and 2, the fraction of firms in industry ω with cost greater than c is*

$$
e^{-(c/C_\omega)^\gamma}
$$

where

$$
\begin{aligned}
C_\omega &= \left\{\sum_{\rho \in \varrho(\omega)} \kappa_{\omega\rho} B_{\omega\rho} \left((t_x^*)^{\alpha_R^\rho} (t_l)^{\alpha_L^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma}\right\}^{-\frac{1}{\gamma}} && (3) \\
t_x^* &= \left(\int_1^\infty t_x^{-\zeta^R} dT(t_x)\right)^{-1/\zeta_R} \\
\kappa_{\omega\rho} &= \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} \mathcal{V}_{\omega\rho}(dv) \prod_{\hat{\omega} \in \hat{\Omega}^\rho} \Gamma\left(1 - \frac{\beta_{\hat{\omega}}^\rho}{\zeta_{\hat{\omega}}}\right) \Gamma\left(1 - \frac{\zeta_{\hat{\omega}}}{\gamma}\right)^{\beta_{\hat{\omega}}^\rho / \zeta_{\hat{\omega}}}
\end{aligned}
$$

Proof. Consider recipe ρ that uses labor and intermediate inputs $\hat{\Omega}^\rho = \{\hat{\omega}_1, ..., \hat{\omega}_n\}$. Let $H_{\omega\rho}(c)$ be the arrival rate of a technique that delivers cost weakly less than c. Then $1 - F_{\omega\rho}(c)$ is the probability that no such techniques arrive, or $e^{-H_{\omega\rho}(c)}$. To find $H_{\omega\rho}(c)$, we consider first a technique of recipe ρ for which the common components of input-augmenting productivities are $b_l, b_1, ..., b_n$. To find the probability that the technique delivers unit cost weakly less than c we integrate over the effective cost of all inputs:

$$
\int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(\lambda_l, \lambda_1, ..., \lambda_n\right) \leq c\right\} \prod_{k=1}^n e^{-\left(\lambda_k b_k / \Lambda_{\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_k
$$

To find $H_{\omega\rho}(c)$, we integrate over the arrival of such techniques:

$$
H_{\omega\rho}(c) = \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(\lambda_l, \lambda_1, ..., \lambda_n\right) \leq c\right\} \left(\prod_{k=1}^n e^{-\left(\lambda_k b_k / \Lambda_{\omega\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\omega\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_k\right) \mathcal{B}_{\omega\rho}(db)
$$

Using the definition of $\lambda_l = \frac{t_l}{b_l}$ and the homogeneity of the cost function, this is

$$
H_{\omega\rho}(c) = \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l c}, \frac{\lambda_1}{c}, ..., \frac{\lambda_n}{c}\right) \leq 1\right\} \left(\prod_{k=1}^n e^{-\left(\lambda_k b_k / \Lambda_{\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_k\right) \mathcal{B}_{\omega\rho}(db)
$$

It will be useful to make the changes of variables $v_k = \lambda_k / c$, $v_l = \frac{t_l}{c b_l}$, and $m_j = (\lambda_l b_l / \Lambda_{\hat{\omega}_l})^{\zeta_{\hat{\omega}_j}}$ to express $H_{\omega\rho}$ as

$$
H_{\omega\rho}(c) = \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} B_{\omega\rho} \left(\frac{t_l}{c}\right)^{-\beta_l^\rho} \left(\prod_{k=1}^n \left(\frac{m_k^{1/\zeta_{\hat{\omega}_k}} \Lambda_{\hat{\omega}_k}}{c}\right)^{-\beta_{\hat{\omega}_k}^\rho} e^{-m_k} dm_k\right) \mathcal{V}_{\omega\rho}(dv)
$$

or more simply,

$$H_{\omega\rho}(c) = \tilde{\kappa}_{\omega\rho} B_{\omega\rho} t_l^{-\beta_l^\rho} \Lambda_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho} ... \Lambda_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho} c^\gamma$$

where

$$\tilde{\kappa}_{\omega\rho} \equiv \int_0^\infty ... \int_0^\infty 1 \left\{ \mathcal{C}_{\omega\rho} \left(v_l, v_1, ..., v_n \right) \leq 1 \right\} \mathcal{V}_{\omega\rho}(dv) \prod_{k=1}^n \int_0^\infty m_k^{-\beta_{\hat{\omega}_k}^\rho / \zeta_{\hat{\omega}_k}} e^{-m_k} dm_k$$

Assumptions 1 and 2e guarantee that $\tilde{\kappa}_{\omega\rho}$ is finite: the first integral is finite because of Lemma F.2, and the subsequent integrals can be written as

$$\int_0^\infty m_{\hat{\omega}_k}^{-\beta_{\hat{\omega}_k}^\rho / \zeta_{\hat{\omega}_k}} e^{-m_k} dm_k = \Gamma \left(1 - \frac{\beta_{\hat{\omega}_k}^\rho}{\zeta_{\hat{\omega}_k}} \right)$$

Using the fact that $1 - F_\omega(c) = \prod_{\rho \in \varrho(\omega)} \left[1 - F_{\omega\rho}(c) \right] = \prod_{\rho \in \varrho(\omega)} e^{-H_{\omega\rho}(c)}$, we have

$$1 - F_\omega(c) = e^{-(c/C_\omega)^\gamma}$$

where C_ω is defined as

$$C_\omega \equiv \left[\sum_{\rho \in \varrho(\omega)} \tilde{\kappa}_{\omega\rho} B_{\omega\rho} t_l^{-\beta_l^\rho} \Lambda_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho} ... \Lambda_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho} \right]^{-1/\gamma} \tag{4}$$

To complete the proof, we will derive expressions for $\Lambda_{\hat{\omega}}$ in terms of $C_{\hat{\omega}}$ and substitute into (4). Note first that for each $\hat{\omega} \in \hat{\Omega}^\rho$ we have

$$\begin{aligned} \int_0^\infty c^{-\zeta_\omega} dF_\omega(c) &= \int_0^\infty c^{-\zeta_\omega} C_\omega^{-\gamma} \gamma c^{\gamma - 1} e^{-(c/C_\omega)^\gamma} dc = C_\omega^{-\zeta_\omega} \int_0^\infty v^{-\frac{\zeta_\omega}{\gamma}} e^{-v} dv \\ &= C_\omega^{-\zeta_\omega} \Gamma \left(1 - \frac{\zeta_\omega}{\gamma} \right) \end{aligned}$$

therefore Lemma F.1 implies

$$\Lambda_{\hat{\omega}} = \begin{cases} t_x^* C_{\hat{\omega}} \Gamma \left(1 - \frac{\zeta_R}{\gamma} \right)^{-1/\zeta_R} & \hat{\omega} \in \hat{\Omega}_R \\ C_{\hat{\omega}} \Gamma \left(1 - \frac{\zeta_H}{\gamma} \right)^{-1/\zeta_H} & \hat{\omega} \in \hat{\Omega}_H \end{cases}$$

Plugging this into (4), defining $\kappa_{\omega\rho} \equiv \tilde{\kappa}_{\omega\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} \Gamma \left(1 - \frac{\zeta_{\hat{\omega}}}{\gamma} \right)^{\beta_{\hat{\omega}}^\rho / \zeta_{\hat{\omega}}}$, and using $\alpha_{\hat{\omega}}^\rho = \frac{\beta_{\hat{\omega}}^\rho}{\gamma}$ gives the result. ■

F.2 Factor Shares

Consider a firm in industry ω. If, in equilibrium, the firm uses a technique of recipe ρ (that uses labor and intermediate inputs $\hat{\Omega}^\rho = \{\hat{\omega}_1, ..., \hat{\omega}_n\}$) with input-augmenting productivities $b = \{b_l, b_1, ..., b_n\}$, effective cost of intermediate inputs $\lambda = \{\lambda_1, ..., \lambda_n\}$, then its payment to supplier of a relationship-specific input $\hat{\omega}_i$ with wedge t_{xs} is

$$\begin{aligned} p_s x_s &= t_{xs}^{-1} \lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda) y_j, \quad \omega_i \in \Omega_R^\rho \\ p_s x_s &= \lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda) y_j, \quad \omega_i \in \Omega_H^\rho \\ wl &= \lambda_l \mathcal{C}_{\omega\rho\hat{\omega}_i} \left(\lambda_l, \lambda_1, ..., \lambda_n \right) y_j + \sum_{\hat{\omega} \in \hat{\Omega}_R^\rho} (1 - t_{xs}^{-1}) \lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda) y_j \end{aligned}$$

where $\mathcal{C}_{\omega\rho l}$ and $\mathcal{C}_{\omega\rho\hat{\omega}}$ denote the partial derivatives of the cost function with respect to the cost of labor and to the cost of input $\hat{\omega}$ respectively.

We characterize average revenue shares of each input in several intermediate steps.

Effective Cost Shares

For any technique of recipe ρ that delivers cost c, let $D_{\omega\rho}(c)$ denote the probability that the technique is actually chosen by the firm.[7]

Lemma F.3 *Under Assumptions 1 and 2, the average effective cost share of the ith intermediate input and of labor among firms that, in equilibrium, use recipe ρ and have unit cost weakly less than c_0 are, respectively,*

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)}{\int_v 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)} \tag{5}$$

$$E\left[\left.\frac{\lambda_l C_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\int_v \frac{v_l C_{\omega\rho l}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)}{\int_v 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)} \tag{6}$$

Proof. For a firm in industry ω, the measure of the arrival rate of techniques of recipe ρ with $b = \{b_l, b_1, ..., b_n\}$ and $\lambda = \{\lambda_1, ..., \lambda_n\}$ that deliver cost weakly less than c_0 is

$$1\left\{\mathcal{C}_{\omega\rho}(\lambda) \leq c_0\right\} \left(\frac{b_1\lambda_1}{\Lambda_{\hat{\omega}_1}}\right)^{\zeta_{\hat{\omega}_1}} \frac{\zeta_{\hat{\omega}_1}}{\lambda_1} e^{-\left(\lambda_1 b_1/\Lambda_{\hat{\omega}_1}\right)^{\zeta_{\hat{\omega}_1}}} d\lambda_1 ... \left(\frac{b_n\lambda_n}{\Lambda_{\hat{\omega}_n}}\right)^{\zeta_{\hat{\omega}_n}} \frac{\zeta_{\hat{\omega}_n}}{\lambda_n} e^{-\left(\lambda_n b_n/\Lambda_{\hat{\omega}_n}\right)^{\zeta_{\hat{\omega}_n}}} d\lambda_n \mathcal{B}_{\omega\rho}(db) \tag{7}$$

Such a technique is actually used by the firm with probability $D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(\lambda)\right)$. To find the density of b and λ among firms in industry ω that choose to use a technique of recipe ρ that delivers cost weakly less than c_0 we simply divide the product of (7) and $D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(\lambda)\right)$ by the integral over all such combinations of input-augmenting productivities and effective cost, so that the conditional expectation of $\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}$ is

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\begin{array}{c}\int_b \int_\lambda \frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} 1\left\{\mathcal{C}_{\omega\rho}(\lambda) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(\lambda)\right) \\ \times \left(\frac{b_1\lambda_1}{\Lambda_{\hat{\omega}_1}}\right)^{\zeta_{\hat{\omega}_1}} \frac{\zeta_{\hat{\omega}_1}}{\lambda_1} e^{-\left(\lambda_1 b_1/\Lambda_{\hat{\omega}_1}\right)} d\lambda_1 ... \left(\frac{b_n\lambda_n}{\Lambda_{\hat{\omega}_n}}\right)^{\zeta_{\hat{\omega}_n}} \frac{\zeta_{\hat{\omega}_n}}{\lambda_n} e^{-\left(\lambda_n b_n/\Lambda_{\hat{\omega}_n}\right)} d\lambda_n \mathcal{B}_{\omega\rho}(db)\end{array}}{\begin{array}{c}\int_b \int_\lambda 1\left\{\mathcal{C}_{\omega\rho}(\lambda) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(\lambda)\right) \\ \times \left(\frac{b_1\lambda_1}{\Lambda_{\hat{\omega}_1}}\right)^{\zeta_{\hat{\omega}_1}} \frac{\zeta_{\hat{\omega}_1}}{\lambda_1} e^{-\left(\lambda_1 b_1/\Lambda_{\hat{\omega}_1}\right)} d\lambda_1 ... \left(\frac{b_n\lambda_n}{\Lambda_{\hat{\omega}_n}}\right)^{\zeta_{\hat{\omega}_n}} \frac{\zeta_{\hat{\omega}_n}}{\lambda_n} e^{-\left(\lambda_n b_n/\Lambda_{\hat{\omega}_n}\right)} d\lambda_n \mathcal{B}_{\omega\rho}(db)\end{array}}$$

Making the change of variables of variables $m_k = \left(\lambda_k b_k/\Lambda_{\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}$ for each k and using the definition $\lambda_l = \frac{t_l}{b_l}$ gives

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\begin{array}{c}\int_b \int_m \frac{\frac{\Lambda_{\hat{\omega}_i}}{b_{\hat{\omega}_i}} m_i^{1/\zeta_{\hat{\omega}_i}} C_{\omega\rho\hat{\omega}_i}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right)}{C_{\omega\rho}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right)} 1\left\{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right) \leq c_0\right\} \\ \times D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right)\right) e^{-m_1} dm_1 ... e^{-m_n} dm_n \mathcal{B}_{\omega\rho}(db)\end{array}}{\begin{array}{c}\int_b \int_m 1\left\{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right) \leq c_0\right\} \\ \times D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right)\right) e^{-m_1} dm_1 ... e^{-m_n} dm_n \mathcal{B}_{\omega\rho}(db)\end{array}}$$

where $\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \left\{\frac{\Lambda}{b} m^{1/\zeta}\right\}\right) = \mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \frac{\Lambda_{\hat{\omega}_i}}{b_i} m_i^{1/\zeta_{\hat{\omega}_i}}, ..., \frac{\Lambda_{\hat{\omega}_n}}{b_n} m_n^{1/\zeta_{\hat{\omega}_n}}\right)$. A further change of variables $v_l = \frac{t_l}{b_l}$ and $v_k = \frac{\Lambda_{\hat{\omega}_k}}{b_k} m_k^{1/\zeta_{\hat{\omega}_k}}$ gives

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\begin{array}{c}\int_v \int_m \frac{v_{\hat{\omega}_i} C_{\omega\rho\hat{\omega}_i}(v)}{C_{\omega\rho}(v)} 1\left\{C_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(C_{\omega\rho}(v)\right) e^{-m_1} dm_1 ... e^{-m_n} dm_n \\ \times B_{\omega\rho} t_l^{-\beta_l^\rho} \left(\Lambda_{\hat{\omega}_1} m_1^{1/\zeta_{\hat{\omega}_1}}\right)^{\beta_{\hat{\omega}_1}^\rho} ... \left(\Lambda_{\hat{\omega}_n} m_n^{1/\zeta_{\hat{\omega}_n}}\right)^{\beta_{\hat{\omega}_n}^\rho} \mathcal{V}_{\omega\rho}(dv)\end{array}}{\begin{array}{c}\int_v \int_m 1\left\{C_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(C_{\omega\rho}(v)\right) e^{-m_1} dm_1 ... e^{-m_n} dm_n \\ \times B_{\omega\rho} t_l^{-\beta_l^\rho} \left(\Lambda_{\hat{\omega}_1} m_1^{1/\zeta_{\hat{\omega}_1}}\right)^{\beta_{\hat{\omega}_1}^\rho} ... \left(\Lambda_{\hat{\omega}_n} m_n^{1/\zeta_{\hat{\omega}_n}}\right)^{\beta_{\hat{\omega}_n}^\rho} \mathcal{V}_{\omega\rho}(dv)\end{array}}$$

[7] While it is not relevant for the proof, it turns out that $D_{\omega\rho}(c) = 1 - F_\omega(c)$, the overall probability that a firm's best technique delivers cost c.

Canceling common terms from the numerator and denominator gives (5). (6) can be derived using identical logic. ■

Lemma F.4

$$\int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv) = \beta^{\rho}_{\hat{\omega}_i} A_{\omega\rho}(c_0) \tag{8}$$

$$\int_v \frac{v_l C_{\omega\rho l}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv) = \beta^{\rho}_{L} A_{\omega\rho}(c_0) \tag{9}$$

where $A_{\omega\rho}(c_0)$ is a constant

Proof. For the proof of this lemma we need some additional notation. Define the function $K_{\omega\rho}(c; c_0) = \int_c^{c_0} \frac{1}{\tilde{c}} D_{\omega\rho}(\tilde{c})\, d\tilde{c}$. Define the function $\psi(v_{-i}, c)$ to be the solution to $C_{\omega\rho}(v_{-i}, \psi(v_{-i}, c)) = c$ if such a solution exists, or take the value of zero no such solution exists, i.e., if the v_{-i} are too large for a solution to exist. Finally, define $\mathcal{V}_{\omega\rho,-i}(v_{-i}) \equiv v_l^{\beta^{\rho}_l} \prod_{k \neq i} v_k^{\beta^{\rho}_{\hat{\omega}_k}}$ and $\mathcal{V}_{\omega\rho,l}(v_{-l}) \equiv \prod_k v_k^{\beta^{\rho}_{\hat{\omega}_k}}$

We can express the left hand side of (8) as

$$\begin{aligned} LHS &= \int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv) \\ &= \int_{v_{-i}} \int_{v_i} \frac{v_i C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{C_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(C_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \mathcal{V}_{\omega\rho,-i}(dv_{-i}) \end{aligned} \tag{10}$$

We can use the definition of $\psi(v_{-i}, c)$, integrate by parts, and then use the definition of $\psi(v_{-i}, c)$ again to express the innermost integral as

$$\begin{aligned} & \int_{v_i} \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\ = & \int_0^{\psi(v_{-i},c_0)} \frac{v_i C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\ = & \int_0^{\psi(v_{-i},c_0)} \frac{C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}} dv_i \\ = & -K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}} \Big|_0^{\psi(v_{-i},c_0)} + \int_0^{\psi(v_{-i},c_0)} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \left(\beta^{\rho}_{\hat{\omega}_i}\right)^2 v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\ = & \int_0^{\psi(v_{-i},c_0)} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \left(\beta^{\rho}_{\hat{\omega}_i}\right)^2 v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\ = & \beta^{\rho}_{\hat{\omega}_i} \int_{v_i} 1\left\{C_{\omega\rho}(v) \leq c_0\right\} K_{\omega\rho}\left(C_{\omega\rho}(v); c_0\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \end{aligned}$$

Plugging this back into (10) gives

$$LHS = \beta^{\rho}_{\hat{\omega}_i} \int_v 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v), c_0\right) \mathcal{V}_{\omega\rho}(dv)$$

The derivation of (9) follows identical logic. ■

Proposition F.2 *Under Assumptions 1 and 2, the average effective cost share of the ith intermediate input and of labor among firms that, in equilibrium, use recipe ρ and have unit cost weakly less than c_0 are,* *respectively,*

$$\begin{aligned} E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] &= \alpha^{\rho}_{\hat{\omega}_i} \\ E\left[\left.\frac{\lambda_l C_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] &= \alpha^{\rho}_{L} \end{aligned}$$

Proof. Lemma F.3 gives

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)}{\int_v 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)}$$

Lemma F.4 implies that the numerator equals $\beta^{\rho}_{\omega_i} A_{\omega\rho}(c_0)$. The homogeneity of the cost function implies that $1 = \frac{v_l C_{\omega\rho l}(v)}{C_{\omega\rho}(v)} + \sum_i \frac{v_i C_{\omega\rho\hat{\omega}_i}(v)}{C_{\omega\rho}(v)}$, so the denominator equals $\beta^{\rho}_{l} A_{\omega\rho}(c_0) + \sum_i \beta^{\rho}_{\hat{\omega}_i} A_{\omega\rho}(c_0) = \gamma A_{\omega\rho}(c_0)$. Together these imply that $E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \alpha^{\rho}_{\hat{\omega}_i}$. Identical logic implies that $E\left[\left.\frac{\lambda_l C_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \alpha^{\rho}_{L}$. ■

Actual Cost Shares

Lemma F.5 *Among firms that produce ω that, in equilibrium, use a supplier for relationship-specific input $\hat{\omega} \in \Omega^{\rho}_{R}$ that delivers effective cost $\lambda_{\hat{\omega}}$, the harmonic average of the wedge is*

$$E\left[t^{-1}_{xs} | \lambda_{\hat{\omega}}\right]^{-1} = \bar{t}_x \equiv \left(\int_1^{\infty} t_x^{-1} d\tilde{T}(t_x)\right)^{-1}$$

where $\tilde{T}(t_x) \equiv \frac{\int_1^{t_x} t^{-\zeta_R} dT(t)}{\int_1^{\infty} t^{-\zeta_R} dT(t)}$.

Proof. Consider all suppliers drawn by j to supply input $\hat{\omega}$ for a technique of recipe ρ with common component of input-augmenting productivity b. The effective cost delivered by a supplier is $\frac{t_{xs} p_s}{z_s b}$ where $p_s = c_s$ is the price charged by the supplier. Given the match-specific productivity z_s and wedge t_{xs}, the probability that the supplier's cost is low enough to deliver an effective cost weakly less than λ is the probability that c_s is small enough to so that $\frac{t_{xs} c_s}{z_s b} < \lambda$, i.e., $c_s < \lambda b z_s / t_{xs}$, or $1 - F_{\hat{\omega}}\left(\lambda b z_s / t_{xs}\right)$. Integrating over possible values of z and t_x, the arrival rate of suppliers that deliver effective cost lower than λ is $\int_0^{\infty} \int_1^{\infty} \left[1 - F_{\hat{\omega}}\left(\lambda b z / t\right)\right] dT(t) \zeta_R z^{-\zeta_R - 1} dz$. Second, the arrival rate of suppliers that deliver effective cost less than λ and for which the wedge is weakly less than t_x is $\int_0^{\infty} \int_1^{t_x} \left[1 - F_{\hat{\omega}}\left(\lambda b z / t\right)\right] dT(t) \zeta_R z^{-\zeta_R - 1} dz$. Together, these imply that, among suppliers who deliver effective cost less than λ, the probability that the wedge is less than t_x is

$$\begin{aligned} \Pr\left(t_{xs} < t_x | \lambda_s \leq \lambda, b\right) &= \frac{\int_0^{\infty} \int_1^{t_x} \left[1 - F_{\hat{\omega}}\left(\lambda b z / t\right)\right] dT(t) z^{-\zeta_R - 1} dz}{\int_0^{\infty} \int_1^{\infty} \left[1 - F_{\hat{\omega}}\left(\lambda b z / t\right)\right] dT(t) z^{-\zeta_R - 1} dz} \\ &= \frac{\int_0^{\infty} \int_1^{t_x} \left[1 - F_{\hat{\omega}}(u)\right] \lambda^{\zeta_R} t^{-\zeta_R} dT(t) u^{-\zeta_R - 1} du}{\int_0^{\infty} \int_1^{\infty} \left[1 - F_{\hat{\omega}}(u)\right] \lambda^{\zeta_R} t^{-\zeta_R} dT(t) u^{-\zeta_R - 1} du} \\ &= \frac{\int_1^{t_x} t^{-\zeta_R} dT(t)}{\int_1^{\infty} t^{-\zeta_R} dT(t)} \\ &= \tilde{T}(t_x) \end{aligned}$$

where the second line uses the change of variables $u = \lambda b z / t$. With this, we have that among suppliers that deliver effective cost weakly greater than λ, the harmonic average of wedges is

$$\bar{t}_x \equiv E\left[t^{-1}_{xs} | \lambda_s \leq \lambda, b\right]^{-1}$$

Since $\bar{t}_x$ does not depend on λ, the expectation must be the same for each λ, b, i.e.,

$$\bar{t}_x = E\left[t_{xs}^{-1}|\lambda_s, b\right]^{-1}$$

Finally, this equation holds regardless of whether s is selected as a supplier. In other words,

$$E\left[t_{xs}^{-1}|\lambda, b\right] = E\left[t_{xs}^{-1}|\lambda, b, s \text{ selected as supplier}\right] = \bar{t}_x^{-1}$$

■

Proposition F.3 *Among firms in industry ω that, in equilibrium, use recipe ρ and have unit cost c:*

- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_R^\rho$ is $\frac{1}{\bar{t}_x}\alpha_{\hat{\omega}}^\rho$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ is $\alpha_{\hat{\omega}}^\rho$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ is $\alpha_L^\rho + (1 - \frac{1}{\bar{t}_x})\alpha_R^\rho$*

Proof. Consider a relationship specific-input $\hat{\omega}$. Note first that the share of j's expenditures spent on $\hat{\omega}$ is $\frac{1}{t_{\hat{\omega}x}} \frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}$. Note that conditional on λ_i, $t_{\hat{\omega}x}$ is independent of any other feature of the firm's sourcing decision. We therefore have, by iterated expectations,

$$\begin{aligned}
E\left[\frac{1}{t_{\hat{\omega}x}} \frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c, \rho\right] &= E\left[\frac{1}{t_{x\hat{\omega}}} \frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c, \rho\right] \\
&= E\left\{E\left[\frac{1}{t_{x\hat{\omega}}} \frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c, \rho, b, \lambda\right]\middle| c, \rho\right\} \\
&= E\left\{E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c, \rho, b, \lambda\right] \frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c, \rho\right\} \\
&= E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c, \rho, b, \lambda\right] E\left\{\frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c, \rho\right\} \\
&= \frac{1}{\bar{t}_x}\alpha_{\hat{\omega}}^\rho
\end{aligned}$$

The expression for homogeneous inputs follows directly from Proposition F.2. The expression for labor follows from the fact that the cost shares sum to 1. ■

Corollary F.1 *Among firms in industry ω that, in equilibrium, use recipe ρ:*

- *the average and aggregate shares of expenditures spent on input $\hat{\omega} \in \Omega_R^\rho$ are both $\frac{1}{\bar{t}_x}\alpha_{\hat{\omega}}^\rho$*
- *the average and aggregate shares of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ are $\alpha_{\hat{\omega}}^\rho$*
- *the average and aggregate shares of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ are $\alpha_L^\rho + (1 - \frac{1}{\bar{t}_x})\alpha_R^\rho$*

Proof. This follows directly from the previous corollary by integrating over realizations of c. ■

We next turn to revenue shares.

Proposition F.4 *Among firms in industry ω that, in equilibrium, use recipe ρ and have unit cost c:*

- *the average share of revenue spent on input $\hat{\omega} \in \Omega_R^\rho$ is $\frac{1}{\bar{t}_x}\alpha_{\hat{\omega}}^\rho E[\frac{c}{p}|c, \rho]$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ is $\alpha_{\hat{\omega}}^\rho E[\frac{c}{p}|c, \rho]$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ is $\left(\alpha_L^\rho + (1 - \frac{1}{\bar{t}_x})\alpha_R^\rho\right) E[\frac{c}{p}|c, \rho]$*

Proof. This proof closely follows the proof of cost shares. Let $\bar{p}_j$ we the average price firm j receives from selling its good. This is a weighted average of c_j, the price paid by other firms that use j as a supplier, and $\frac{\varepsilon}{\varepsilon-1}c_j$, the price paid by the household. Consider a relationship-specific input $\hat{\omega}$. j's revenue share of input $\hat{\omega}$ is $\frac{c}{\bar{p}_j}\frac{1}{t_{\hat{\omega}x}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}$. Note that conditional on c_j, $\bar{p}_j$ is independent of any feature of the firm's sourcing decision, and conditional on λ_i, $t_{\hat{\omega}x}$ is independent of any other feature of the firm's sourcing decision. Putting the pieces together, we have

$$\begin{aligned}
E\left[\frac{c}{\bar{p}}\frac{1}{t_{\hat{\omega}x}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right] &= E\left[\frac{c}{\bar{p}}\frac{1}{t_{x\hat{\omega}}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right] \\
&= E\left\{E\left[\frac{c}{\bar{p}}\frac{1}{t_{x\hat{\omega}}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho,b,\lambda\right]\middle| c,\rho\right\} \\
&= E\left\{E\left[\frac{c}{\bar{p}}\middle| c,\rho,b,\lambda\right]E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c,\rho,b,\lambda\right]\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right\} \\
&= E\left[\frac{c}{\bar{p}}\middle| c,\rho,b,\lambda\right]E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c,\rho,b,\lambda\right]E\left\{\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right\} \\
&= E\left[\frac{c}{\bar{p}}\middle| c,\rho\right]\frac{1}{\bar{t}_x}\alpha^\rho_{\hat{\omega}}
\end{aligned}$$

The derivation for homogeneous inputs is the same but without the wedge (i.e., setting the wedge to 1). The derivation for labor is similar. ■

Corollary F.2 *Among firms in industry ω that, in equilibrium use recipe ρ, the following expressions hold:*

$$\begin{aligned}
0 &= E\left[\frac{s_{jR}}{\frac{\alpha^\rho_R}{t_x}} - \frac{s_{jH}}{\alpha^\rho_H}\middle|\rho\right] \\
0 &= E\left[\frac{s_{jR}+s_{jL}}{\alpha^\rho_R+\alpha^\rho_L} - \frac{s_{jH}}{\alpha^\rho_H}\middle|\rho\right]
\end{aligned}$$

Proof. This follows from rearranging the expressions in the previous proposition and integrating over c. ■

F.3 Counterfactuals

Given the household's preferences, let P_ω be the ideal price index for the consumption aggregate for industry ω and let P be the overall ideal price index. These satisfy $P_\omega \equiv \left(\int_0^{J_\omega} p_j^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ and $P = \left(\sum_\omega \upsilon_\omega P_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}}$.

Since each firm charges a fixed markup over marginal cost in sales to the household, firm j in industry ω would charge a price of $p_j = \frac{\varepsilon}{\varepsilon-1}c_j$. Thus the price index for ω satisfies

$$P_\omega^{1-\varepsilon} = \int_0^{J_\omega}\left(\frac{\varepsilon}{\varepsilon-1}c_j\right)^{1-\varepsilon}dj = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}J_\omega\int_0^\infty c^{1-\varepsilon}dF_\omega(c)$$

Proposition 1 gives $F_\omega(c) = 1 - e^{-(c/C_\omega)^\gamma}$. Integrating yields $\int_0^\infty c^{1-\varepsilon}dF_\omega(c) = \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)C_\omega^{1-\varepsilon}$, which implies that the industry price index can be expressed as

$$P_\omega = \frac{\varepsilon}{\varepsilon-1}J_\omega^{\frac{1}{1-\varepsilon}}\Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{1}{1-\varepsilon}}C_\omega$$

The overall price index is therefore

$$P = \frac{\varepsilon}{\varepsilon-1}\left(\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}}\Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}}C_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

To find total profit, note that j's profit (which comes only from sales to the household because firm-to-firm sales are priced at marginal cost) is $\pi_{\omega j} = u_{\omega j}(p_{\omega j} - c_{\omega j})$. Using $u_{\omega j} = U\upsilon_\omega \left(\frac{P_\omega}{P}\right)^{-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon} = UP\upsilon_\omega \left(\frac{P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon} \frac{1}{p_j}$ and $c_{\omega j} = \frac{\varepsilon-1}{\varepsilon} p_{\omega j}$, this is $\pi_{\omega j} = UP\upsilon_\omega \left(\frac{P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon} \frac{1}{\varepsilon}$. Total profit can be found by integrating over all firms

$$\Pi = \sum_\omega \int_0^{J_\omega} \pi_{\omega j} dj = \sum_\omega \int_0^{J_\omega} UP \left(\frac{\upsilon_\omega P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon} \frac{1}{\varepsilon} dj = \frac{1}{\varepsilon} UP$$

Total household income is profit plus wage income, while its expenditure is UP, so its budget is $UP = \Pi + wL = \frac{1}{\varepsilon}UP + wL$, or (normalizing the wage to unity)

$$U = \frac{\varepsilon}{\varepsilon - 1} \frac{w}{P} L = \left(\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma \left(1 - \frac{\varepsilon - 1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta} \right)^{\frac{1}{\eta-1}} L$$

Proposition F.5 *A change in the distribution of relationship-specific intermediate input wedges from T to T' leads to a change in household utility that can be summarized by*

$$\frac{U'}{U} = \left(\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta} \right)^{\frac{1}{\eta-1}} \tag{11}$$

and the change in industry cost indexes satisfy the following system of equations

$$\frac{C'_\omega}{C_\omega} = \left[\sum_{\rho\in\varrho(\omega)} R_{\omega\rho} \left(\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho} \prod_{\hat\omega\in\hat\Omega_\rho} \left(\frac{C'_{\hat\omega}}{C_{\hat\omega}}\right)^{\alpha_{\hat\omega}^\rho} \right)^{-\gamma} \right]^{-\frac{1}{\gamma}} \tag{12}$$

Proof. The share of the household's spending on goods from ω is

$$HH_\omega = \frac{\upsilon_\omega P_\omega^{1-\eta}}{P^{1-\eta}} = \frac{\upsilon_\omega \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\eta} J_\omega^{\frac{1-\eta}{1-\varepsilon}} \Gamma \left(1 - \frac{\varepsilon-1}{\gamma}\right)^{\frac{1-\eta}{1-\varepsilon}} C_\omega^{1-\eta}}{P^{1-\eta}} \tag{13}$$

Using $U = \frac{\varepsilon}{\varepsilon-1} L/P$ and rearranging gives

$$HH_\omega = \frac{\upsilon_\omega J_\omega^{\frac{1-\eta}{1-\varepsilon}} \Gamma \left(1 - \frac{\varepsilon-1}{\gamma}\right)^{\frac{1-\eta}{1-\varepsilon}} C_\omega^{1-\eta}}{(U/L)^{\eta-1}}$$

Under the counterfactual, we have

$$\begin{aligned}
(U'/L)^{\eta-1} &= \sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma \left(1 - \frac{\varepsilon - 1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} (C'_\omega)^{1-\eta} \\
&= \sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma \left(1 - \frac{\varepsilon - 1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta} \\
&= \sum_\omega HH_\omega (U/L)^{\eta-1} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}
\end{aligned}$$

Rearranging gives (11).

We next show that the share of revenue is $R_{\omega\rho} = \kappa_{\omega\rho} B_{\omega\rho} \left(\frac{t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat\omega\in\hat\Omega\rho} C_{\hat\omega}^{\alpha_{\hat\omega}^\rho}}{C_\omega} \right)^{-\gamma}$, where we define

$C_{\omega\rho} \equiv \kappa_{\omega\rho}\left(t_l^{\alpha_L^\rho}(t_x^*)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}^\rho}(C_{\hat{\omega}})^{\alpha_{\hat{\omega}}^\rho}\right)^\gamma$, so that $F_{\omega\rho}(c) = 1 - e^{-(c/C_{\omega\rho})^\gamma}$. This follows from two facts. First, among producers of ω that have efficiency q, the fraction that use recipe ρ is:

$$\begin{aligned}
\Pr(\rho|c,\omega) &= \frac{\left(\prod_{\tilde{\rho}\neq\rho}[1-F_{\omega\tilde{\rho}}(c)]\right)F'_{\omega\rho}(c)}{F'_\omega(c)} = \frac{F'_{\omega\rho}(c)/\left[1-F_{\omega\rho}(c)\right]}{F'_\omega(c)/[1-F_\omega(c)]} \\
&= \kappa_{\omega\rho}B_{\omega\rho}\left(\frac{t_l^{\alpha_L^\rho}(t_x^*)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}^\rho}C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}}{C_\omega}\right)^{-\gamma}
\end{aligned}$$

where the last equality follows from $F_{\omega\rho}(c) = 1 - e^{-\kappa_{\omega\rho}B_{\omega\rho}\left(t_l^{\alpha_L^\rho}(t_x^*)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}^\rho}C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma}c^\gamma}$ and $F_\omega(c) = 1 - e^{-(c/C_\omega)^\gamma}$. The second fact is that, conditional on c, revenue is independent of the recipe chosen by a firm or any other feature of the firm's sourcing decisions.

Finally, we have that the counterfactual industry cost indexes $\{C'_\omega\}$ satisfy

$$\begin{aligned}
C'_\omega &= \left\{\sum_{\rho\in\varrho(\omega)}\kappa_{\omega\rho}B_{\omega\rho}\left(t_l^{\alpha_L^\rho}(t_x^{*\prime})^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}^\rho}(C'_{\hat{\omega}})^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma}\right\}^{-\frac{1}{\gamma}} \\
&= \left\{\sum_{\rho\in\varrho(\omega)}\kappa_{\omega\rho}B_{\omega\rho}\left(t_l^{\alpha_L^\rho}(t_x^*)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}^\rho}(C_{\hat{\omega}})^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma}\left(\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma}\right\}^{-\frac{1}{\gamma}} \\
&= \left\{\sum_{\rho\in\varrho(\omega)}R_{\omega\rho}C_\omega^{-\gamma}\left(\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma}\right\}^{-\frac{1}{\gamma}}
\end{aligned}$$

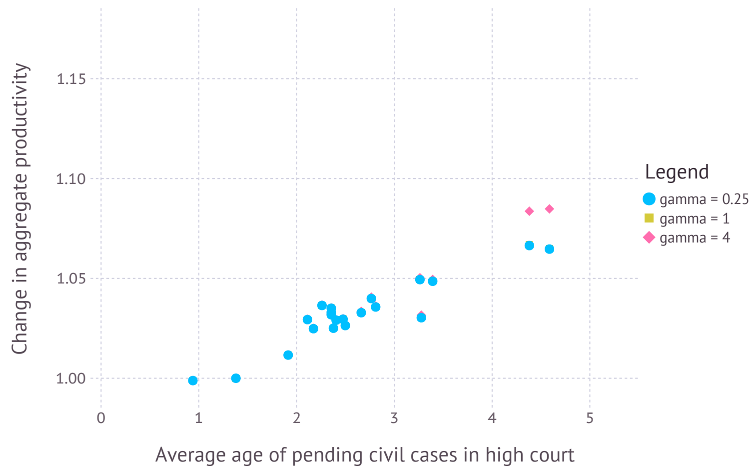
Rearranging yields (12). ■

G Additional Structural Results

G.1 Counterfactual: Robustness to different parameter values

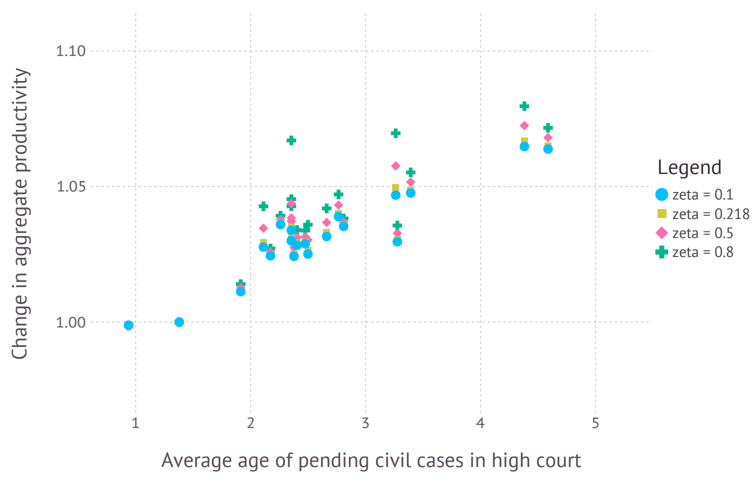
Figures G.1 and G.2 show the results from the welfare counterfactual for different values of γ and ζ.

Figure G.1 Welfare counterfactual for different elasticities γ



The figure shows the counterfactual increase in U for each state for different values of γ. For $\Delta U < 5\%$ the differences are so small that the markers are overlapping.

Figure G.2 Welfare counterfactual for different elasticities ζ



The figure shows the counterfactual increase in U for each state for different values of ζ.

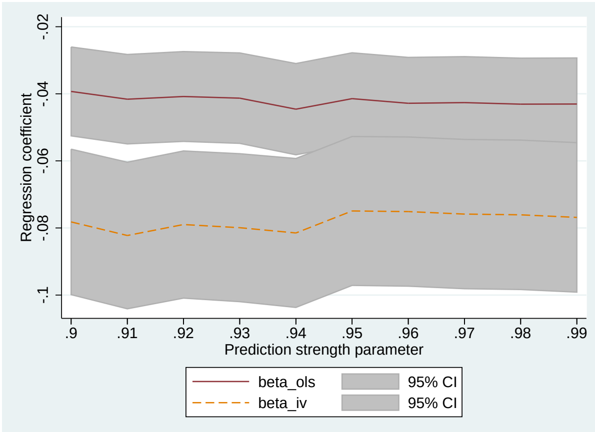
G.2 Fineness of recipes

This subsection explores the robustness of our estimates to the choice of how finely to define recipes. In our clustering procedure that defines recipes we use the prediction strength method of Tibshirani and Walther (2005) to determine the number of clusters in each industry. Similar to cross-validation, the prediction strength method divides the sample into two subsamples (A and B) and assesses the predictive power of clusters obtained from each subsample. In our benchmark implementation that we use for the results in the paper, we choose a threshold parameter of 0.95. Here, we explore how much this choice matters. To do so, we run linear regressions that most closely mimic the structural regressions (cf. the GMM moment conditions in Proposition 5):

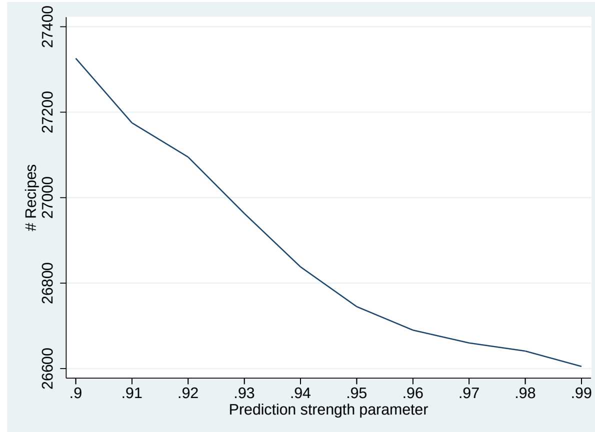
$$\log\left(\frac{\overline{s}_R^{\rho d}}{\overline{s}_H^{\rho d}}\right) = \beta \cdot (\text{Court quality})_d + \nu_\rho + \varepsilon_{\rho d}$$

where $\overline{s}_R^{\rho d}$ (and $\overline{s}_H^{\rho d}$) is the weighted average sales share of relationship-specific (homogeneous) inputs of plants that produce using recipe ρ in state d (weights are the probability weights as in the GMM procedure), and ν_ρ is a set of fixed effects. The estimate for β has a negative sign: Among plants that use a particular recipe, the sales shares of relationship-specific inputs is low compared to homogeneous inputs when courts are slow (i.e., when the average age of pending cases is high). The same holds when we instrument for court quality with the log age of court.

Figure G.2 shows that the point estimates for the regression coefficients do not change much with the threshold parameter. This is because the procedure identifies a similar number of recipes for a relatively broad range of threshold parameters.



(a) Regression Coefficients



(b) Number of Recipes

Figure G.3 Regression coefficients & number of recipes for different levels of recipe fineness

G.3 Identifying Recipes and Distortions: Monte Carlo Exercises

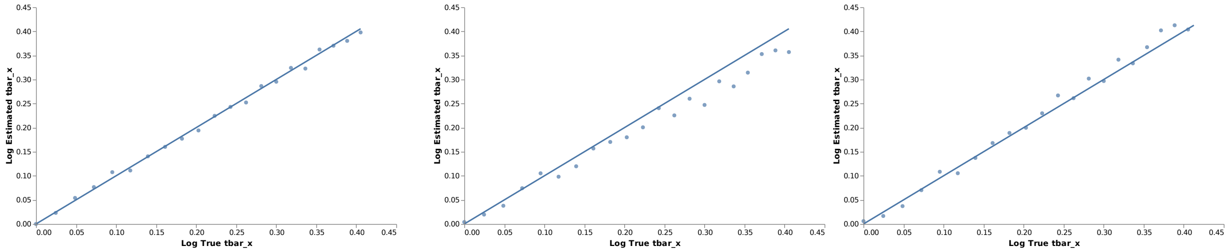
An important part of our exercise is to separate recipes from distortions. Fundamentally, identification is provided by the assumption that recipe production functions are invariant across states, whereas distortion parameters τ_d are state-specific. At the same time, our quantitative exercise also relies on a way to assign plants to recipes (which we do using the clustering algorithm), and on a way to determine the number of recipes in that procedure.

In this section, we explore the implications of the choice of the number of recipes, i.e. the number of clusters in the clustering procedure. We first discuss what could go wrong when choosing the "wrong" number of clusters, and illustrate these considerations with Monte Carlo simulations of small economies of our model. If the number of recipes we allow for is sufficiently large or the choice of recipes is uncorrelated with distortions, our estimator is likely to be consistent. We then perform a Monte Carlo study to assess the small-sample properties of our estimator (for the sample size we have).

G.3.1 In large samples

Our problem is closely related to the group fixed effect estimator of Bonhomme and Manresa (2015). Bonhomme and Manresa study the asymptotic properties (including consistency) of an estimator that estimates simultaneously group memberships, group fixed effects, and coefficients on observable characteristics in a linear panel model. In fact, our moment conditions can be mapped into a extension of their model (see equation 7 of their paper). Bonhomme and Manresa propose an iterative algorithm similar to ours, and show that if the number of groups is correct, the estimation of the covariates is consistent. In our context, this would mean that if we have the right number of recipes, our estimates of the distortions would be consistent. There are some small differences between our approach and that of Bonhomme and Manresa, so we use Monte Carlo simulations to confirm that this property does indeed seem to hold in our model (see discussion below).

To think about the properties of our estimator under the wrong number of clusters, we draw from another closely related paper, Moon and Weidner (2015), who study least squares estimators in panel models with a factor structure with an unknown number of factors, as well as the discussion in section S3 of the supplementary Appendix in Bonhomme and Manresa (2015), which also draws on Moon and Weidner (2015). In those models, if one allows for too few groups/factors, estimates of the distortions may not be consistent, whereas if one allows for too many groups/factors, the estimates of the distortions will be consistent, although this comes at the cost of reduced power. Our model is very similar, but does not map exactly into that of Moon and Weidner, so we cannot apply their analytical results directly to our context. Nevertheless, Monte Carlo simulations of our model confirm that these properties give a good description of the performance of



(a) One cluster per product (b) Two clusters per product (c) Four clusters per product

Figure G.4 MC results: Number of observations not skewed across states

The simulated economy has two products, one which is relationship-specific, the other one homogeneous. For each product there are two recipes; one intensive in the homogeneous good, the other one in the relationship-specific good. There are 21 states, each having the same number of producers using each recipe and producing each product, with distortions $\bar{t}$ as indicated on the horizontal axis. The average estimated $\bar{t}$ from large samples is shown on the vertical axis.

our estimator.

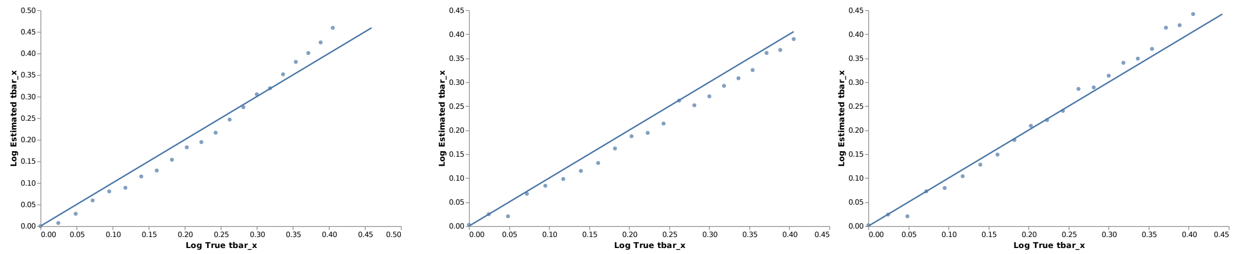
The bottom line is that if the number of recipes we allow for is sufficiently large (greater or equal than the actual number of recipes), our estimator is likely to be consistent. Allowing for too many recipes, however, comes at the cost of precision, as the $\bar{t}_x$ are estimated from within-recipe across-state variation in the shares of materials expenditure in sales, and with more recipes, there are on average fewer observations per recipe.

Here is the intuition: imagine a world with two recipes, one relying more on relationship-specific inputs ("R-inputs") than the other, and two states, one with no distortions and one for which R-inputs are distorted. Assume that we posited a single recipe, i.e., we assign all plants as belonging to the same recipe. If the number of producers using each recipe was the same across states, our estimator would still be consistent: the *average* plant in the distorted state would have a lower cost share on R than the *average* plant in the undistorted state by exactly as much as the distortion (because the weights in the averages are the same).[8] On the other hand, if the more distorted state had relatively more firms using the low-R recipe (perhaps due to technology differences), then the assumption of a single recipe would lead us to overstate the distortions: some of the difference in cost shares on R is due to a different recipe mix, but the estimator would attribute it to larger distortions.

In contrast, if we allow for more recipes than present in the data, the "extra" identified recipe centers will be chosen in a way that is (asymptotically) orthogonal to the terms that identify the distortion parameters $\bar{t}_x$ (this is a formal result in Moon and Weidner (2015), which, based on our Monte Carlo's, seems to extend to our setup). We do, however, lose power, since we identify distortions from variation within recipes across states, and more recipes mean on average fewer observations in each recipe. In the extreme case in which there are more recipes than plants (or, a bit less extreme, if no recipe is used in more than one state), distortions are obviously not identified anymore.

We illustrate this intuition using Monte Carlo experiments on large samples drawn from simulated small model economies. Each model economy consists of two products (one R, one H) and two recipes for each product (one intensive in the R good, the other one intensive in the H good). States vary in their distortions. In Figure G.4 the number of plants using each recipe is the same across states; in Figure G.5 states with larger distortions also happen to have fewer plants using the R-intensive recipe. We find that allowing for too few recipes (one, instead of two) leads to bias in the estimate of the distortions when recipe usage is skewed across states, but not when recipe usage is the same across states. With the right number of recipes (two) or with too many recipes (four instead of two), no such difference appears.

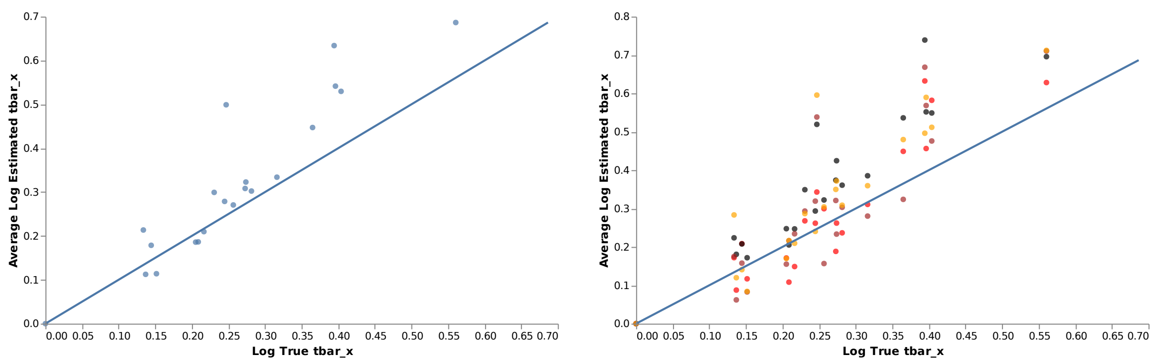
[8] Mathematically, let $s_{\rho_1}^{R,d_1} = \alpha_1/\bar{t}$, $s_{\rho_2}^{R,d_1} = \alpha_2/\bar{t}$, $s_{\rho_1}^{R,d_2} = \alpha_1$, $s_{\rho_2}^{R,d_2} = \alpha_2$, then $w_i^{d_1} = w_i^{d_2}$ implies $\sum_i w_i^{d_1} s_{\rho_i}^{R,d_1} / \sum_i w_i^{d_2} s_{\rho_i}^{R,d_2} = 1/\bar{t}$.



(a) One cluster per product (b) Two clusters per product (c) Four clusters per product

Figure G.5 MC results: Number of observations skewed across states

Setup as above, but here states that are more distorted (higher $\bar{t}$) have relatively more producers using the H-intensive recipe for both products (the most distorted state has about 45% more producers using H-intensive recipes than the least distorted state) .



(a) Average of four MC simulations (b) All four MC simulation results

Figure G.6 MC results using actual number of observations and estimated $\bar{t}$

The figure shows actual (horizontal axis) vs. estimated (vertical axis) distortions from a simulated model economy, where the parameters are the point estimates from our benchmark estimation and the number of simulated plants is the same as in our actual dataset. The left panel shows average estimated distortions across four runs, the right panel shows estimates from each individual run (coded in four different colors).

G.3.2 In small samples

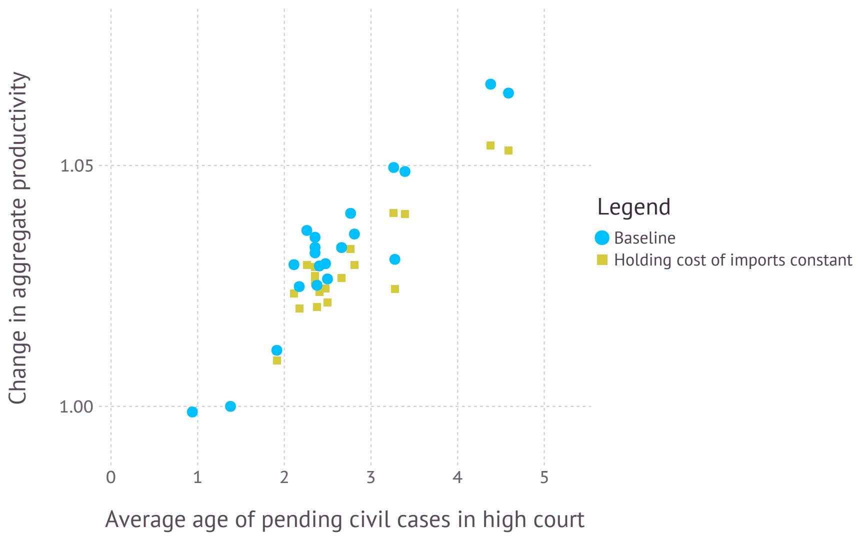
To assess the small-sample properties of our estimation procedure, we conduct a Monte Carlo simulation exercise where we use the actual number of observations in each state and recipe in our data. We simulate four datasets using the point estimates from our baseline results, and run the estimation and classification procedure on these simulated data.[9] Figure G.6 shows the resulting estimates of $\bar{t}$. The left panel shows the average of the estimated $\bar{t}$ over the four runs. The small-sample bias of our estimation procedure seems to be relatively small, in particular for smaller distortions. Larger distortions may be upward biased to some extent. The right panel shows the estimates of each run coded in different colors. Estimates are relatively similar across the four runs, suggesting that the variance of our estimator is not very large.

G.4 Counterfactual with imports from other states

Our benchmark model economy is closed. In this subsection we explore departures from this assumption. Unfortunately, there is little publicly available data on trade across states, in particular for trade in interme-

[9]We do this only four times because each run takes about two days on our server.

Figure G.7 Counterfactual with fixed cost distribution of imports



diate inputs. Van Leemput (2016) pieces together several datasets that cover interstate trade for a number of commodities and modes of transport (based on estimates from the Directorate General of Commercial Intelligence and Statistics). She estimates that on average (across states) imports of manufacturing goods are about 10% of domestic production (less for agriculture). Unfortunately, we do not know what fraction of that is trade in intermediate inputs.

In Figure G.7, we report productivity gains from a counterfactual where we reduce court speed to the level enjoyed by the fastest court (as in the benchmark), but assume that $\frac{10\%}{10\%+100\%}$ of suppliers of each intermediate input is sourced from outside of the state. When reducing contracting frictions, we hold constant the cost distribution of those imports. As we would expect, the gains from reducing contracting frictions are a bit smaller than in our baseline counterfactual.

In our view, reality likely lies between the baseline counterfactual and this alternative. Producers from neighboring states may, in fact, experience cost reductions if contracting frictions are reduced for any producer in their supply chains. Hence, we believe this counterfactual provides a lower bound for the gains from a unilateral reduction in distortions in one state.

G.5 Entry

Suppose there is a representative entrepreneur that can choose the measure of firms in each industry according to a constant elasticity of transformation technology. The mass of firms in each industry $\{J_\omega\}_{\omega\in\Omega}$ must satisfy the constraint $\left(\sum_\omega (J_\omega/h_\omega)^{\frac{1+\beta}{\beta}}\right)^{\frac{\beta}{1+\beta}} \leq 1$, where h_ω indexes the ease of setting up firms in industry ω and β is an elasticity capturing diminishing returns to entering in any particular industry. This specification nests exogenous entry at the extreme of $\beta = 0$ and free entry at $\beta = \infty$. After entry, firms in each industry are ex-ante identical. Following entry, all firms draw techniques and then production occurs.

In equilibrium, let $\bar{\pi}_\omega$ be the average profit of firms in industry ω. The representative entrepreneur takes $\{\bar{\pi}_\omega\}_{\omega\in\Omega}$ as given when making entry decisions, and therefore maximizes expected profit

$$\max_{\{J_\omega\}} \sum_\omega J_\omega \bar{\pi}_\omega \text{ subject to } \left(\sum_\omega (J_\omega/h_\omega)^{\frac{1+\beta}{\beta}}\right)^{\frac{\beta}{1+\beta}} \leq 1$$

The entry choice is

$$J_\omega = h_\omega \left(\frac{(h_\omega \bar{\pi}_\omega)^{1+\beta}}{\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}} \right)^{\frac{\beta}{1+\beta}}$$

We next find an expression for average profit in industry ω. $\bar{\pi}_\omega$. We have assumed that prices in buyer-supplier relationships are set at the supplier's marginal cost. This means that firms make profit only from sales to the final consumer. Since price for firm j in industry ω is $p_{\omega j} = \frac{\varepsilon}{\varepsilon-1} c_{\omega j}$, and final demand is $u_{\omega j} = U_\omega \left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon} = \upsilon_\omega U \left(\frac{P_\omega}{P}\right)^{-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon}$, its profit is $\pi_{\omega j} = (p_{\omega j} - c_{\omega j}) u_{\omega j} = \frac{1}{\varepsilon} p_{\omega j} u_{\omega j} = \frac{1}{\varepsilon} \upsilon_\omega PU \left(\frac{P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}$. Total profit among firms in industry ω is then

$$\bar{\pi}_\omega J_\omega = \int \frac{1}{\varepsilon} \upsilon_\omega PU \left(\frac{P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon} dj = \frac{1}{\varepsilon} \upsilon_\omega PU \left(\frac{P_\omega}{P}\right)^{1-\eta}$$

In equilibrium, the fraction of firms in industry ω with cost greater than c is $e^{-(c/C_\omega)^\gamma}$. Integrating over possible cost realizations gives

$$\begin{aligned} P_\omega^{1-\varepsilon} &= \int \left(\frac{\varepsilon}{\varepsilon-1} c_{\omega j}\right)^{1-\varepsilon} dj = J_\omega \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \int c^{1-\varepsilon} \gamma c^{\gamma-1} C_\omega^{-\gamma} e^{-(c/C_\omega)^\gamma} dc \\ &= \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \Gamma\left(1 - \frac{\varepsilon-1}{\gamma}\right) J_\omega C_\omega^{1-\varepsilon} \end{aligned}$$

Putting these together, average profit is

$$\bar{\pi}_\omega = \frac{1}{\varepsilon} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\eta} \Gamma\left(1 - \frac{\varepsilon-1}{\gamma}\right)^{\frac{1-\eta}{1-\varepsilon}} P^\eta U \upsilon_\omega J_\omega^{\frac{1-\eta}{1-\varepsilon}-1} C_\omega^{1-\eta}$$

Claim G.1 *The mass of firms in industry ω satisfies*

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(h_\omega^{\frac{1-\eta}{1-\varepsilon}} \upsilon_\omega C_\omega^{1-\eta}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \left(h_{\omega'}^{\frac{1-\eta}{1-\varepsilon}} \upsilon_{\omega'} C_{\omega'}^{1-\eta}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Proof. We first rearrange the expression for the mass of firms and then use the expression for average profit. The choice of entry in industry ω is $\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{(h_\omega \bar{\pi}_\omega)^{1+\beta}}{\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}}$, which can be rearranged as

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(A_\omega (J_\omega/h_\omega)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}}{\sum_{\omega'} \left(A_{\omega'} (J_{\omega'}/h_{\omega'})^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}}$$

where $A_\omega \equiv h_\omega^{\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega / J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)$. We want to solve for the denominator. To do this, we can rearrange this further as

$$1 = \frac{A_\omega^{1+\beta} (J_\omega/h_\omega)^{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta) - \frac{1+\beta}{\beta}}}{\sum_{\omega'} \left(A_{\omega'} (J_{\omega'}/h_{\omega'})^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}}$$

or

$$A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}} = \frac{\left[A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right]^{1+\beta}}{\left[\sum_{\omega'}\left(A_{\omega'}\left(J_{\omega'}/h_{\omega'}\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}\right]^{\frac{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta)}{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta)-\frac{1+\beta}{\beta}}}}$$

Summing across ω and then simplifying the right hand side gives

$$\sum_\omega A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}} = \left[\sum_\omega A_\omega^{1+\beta}\left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta)}\right]^{\frac{1}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}$$

or more simply

$$\sum_\omega \left(A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta} = \left(\sum_\omega A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}\right)^{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}$$

We therefore have $\sum_{\omega'}\left(h_{\omega'}\bar{\pi}_{\omega'}\right)^{1+\beta} = \left(\sum_\omega A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}\right)^{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}$ and $\left(h_\omega\bar{\pi}_\omega\right)^{1+\beta} = \left[A_\omega\left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right]^{1+\beta}$. so that

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left[A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right]^{1+\beta}}{\left(\sum_{\omega'} A_{\omega'}^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}\right)^{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}$$

So that solving for J_ω/h_ω gives

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}}{\sum_{\omega'} A_{\omega'}^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}} = \frac{A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Finally, The expression for profits gives $A_\omega \equiv h_\omega^{\frac{1-\eta}{1-\varepsilon}}\left(\bar{\pi}_\omega/J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right) \propto h_\omega^{\frac{1-\eta}{1-\varepsilon}} v_\omega C_\omega^{1-\eta}$, yields the result. ∎

We next show how we can solve for counterfactuals

Claim G.2 *When costs change, the change in the mass of firms in industry ω is*

$$\left(\frac{J'_\omega}{J_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(C'_\omega/C_\omega\right)^{\frac{1-\eta}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_\omega HH_\omega \left(C'_\omega/C_\omega\right)^{\frac{1-\eta}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Proof. Again, using $A_\omega \equiv h_\omega^{\frac{1-\eta}{1-\varepsilon}}\left(\bar{\pi}_\omega/J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right) \propto h_\omega^{\frac{1-\eta}{1-\varepsilon}} v_\omega C_\omega^{1-\eta}$, we have $\frac{A'_\omega}{A_\omega} = \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}$. We also have

$$\left(\frac{J'_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(A'_\omega\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'}\left(A'_{\omega'}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}} = \frac{A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}\left(\frac{A'_\omega}{A_\omega}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}\left(\frac{A'_{\omega'}}{A_{\omega'}}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

So dividing by $\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$ gives

$$\left(\frac{J'_\omega}{J_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(\frac{A'_\omega}{A_\omega}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \frac{A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega''} A_{\omega''}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}} \left(\frac{A'_{\omega'}}{A_{\omega'}}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Finally, using $J_\omega = h_\omega \left(\frac{(h_\omega \bar{\pi}_\omega)^{1+\beta}}{\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}}\right)^{\frac{\beta}{1+\beta}}$ implies $h_\omega = \left(\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}\right)^{\frac{\beta}{(1+\beta)^2}} J_\omega^{\frac{1}{\beta+1}} \bar{\pi}_\omega^{-\frac{\beta}{\beta+1}}$, we have

$$\begin{aligned}
A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} &= \left[h_\omega^{\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega / J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)\right]^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} = \left[\left(\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}\right)^{\frac{\beta}{(1+\beta)^2}\frac{1-\eta}{1-\varepsilon}} J_\omega^{\frac{1}{\beta+1}\frac{1-\eta}{1-\varepsilon}} \bar{\pi}_\omega^{-\frac{\beta}{\beta+1}\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega / J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)\right]^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} \\
&= \left(\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}\right)^{\frac{1}{1+\beta}\frac{\frac{\beta}{1+\beta}\frac{1-\eta}{1-\varepsilon}}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} J_\omega \bar{\pi}_\omega
\end{aligned}$$

Thus we have

$$\left(\frac{J'_\omega}{J_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{(C'_\omega / C_\omega)^{\frac{1-\eta}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \frac{J_{\omega'} \bar{\pi}_{\omega'}}{\sum_{\omega''} J_{\omega''} \bar{\pi}_{\omega''}} (C'_{\omega'} / C_{\omega'})^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Finally, since markups are the same across sectors, we have that $\frac{J_{\omega'} \bar{\pi}_{\omega'}}{\sum_{\omega''} J_{\omega''} \bar{\pi}_{\omega''}} = HH_\omega$, giving the result. ■

Finally, we show how to use this to compute the aggregate counterfactual

Claim G.3 *The change in aggregate productivity of the manufacturing sector is*

$$\frac{U'}{U} = \left\{\sum_{\omega \in \Omega} HH_\omega \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}}$$

Proof. Utility can be expressed as

$$\begin{aligned}
U &= \left\{\sum_{\omega \in \Omega} \Gamma\left(1 - \frac{\varepsilon - 1}{\gamma}\right) \nu_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}\right\}^{\frac{1}{\eta-1}} \\
U' &= \left\{\sum_{\omega \in \Omega} \Gamma\left(1 - \frac{\varepsilon - 1}{\gamma}\right) \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta} \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}} \\
\frac{U'}{U} &= \left\{\sum_{\omega \in \Omega} \frac{\Gamma\left(1 - \frac{\varepsilon-1}{\gamma}\right) \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}}{\sum_{\omega \in \Omega} \Gamma\left(1 - \frac{\varepsilon-1}{\gamma}\right) \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}} \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}} \\
&= \left\{\sum_{\omega \in \Omega} HH_\omega \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}}
\end{aligned}$$

■

This discussion leaves out a few channels. First: we allow for changes in the rates of entry across industries, but we do not allow for changes in the total entry rate to increase. While it would be easy to relax this, we think it is reasonable starting point because if entry costs are denominated in labor (as suggested by Bollard, Klenow and Li (2016)) then the total rate of entry would be invariant to changes in aggregate productivity, which raise both the payoff to and opportunity cost of starting a firm by the same proportion.

We also have ignored any feedback in the change in the number of firms on the number of potential suppliers drawn to provide a good for a technique. It is possible that the mass of such suppliers would increase with the measure of firms in the industry.

H Alternative Distortions

In the model presented in the main text, if firm uses a supplier that shirks on quality of the inputs because of contracting friction, the buyer uses labor to customize the good herself. This increased the total amount of labor used at all stages of production to produce the firm's good (by the firm, its suppliers, its suppliers' suppliers, etc). In this section, we take inspiration from Hsieh and Klenow (2009) and assume that when a firm draws a supplier of a relationship-specific input, she also draws a random wedge t_x from a distribution $T(t_x)$ that is independent of the supplier's cost. The buyer then behaves as if it must pay a tax at rate $t_x - 1$ on expenditures on that input. Like in the baseline, a firm's shadow cost of each input might differ from what it pays to the supplier. Here, however, the firm's shadow unit cost is larger than its actual expenditure on inputs.

Following the notation of the baseline model presented in Section 3, the effective shadow cost of using a supplier with input-augmenting productivity $z_s b_{\hat{\omega}}(\phi)$ that charges price p_s in the presences of the wedge t_{xs} would be $\frac{t_{xs} p_s}{b_{\hat{\omega}}(\phi) z_s}$. j's effective shadow cost of input $\hat{\omega}$ for technique ϕ is the minimum across all potential suppliers:

$$\lambda_{\hat{\omega}}(\phi) = \min_{s \in S_{\hat{\omega}}(\phi)} \frac{t_{xs}(\phi) p_s}{b_{\hat{\omega}}(\phi) z_s(\phi)}.$$

Similarly, the effective shadow cost of labor when using technique ϕ is $\lambda_l(\phi) = \frac{t_l w}{b_l(\phi)}$. For the remainder, we normalize the wage to unity, $w = 1$.

The shadow unit cost delivered by a technique depends on the effective shadow cost of each input. j's shadow cost of producing one unit of output using technique ϕ would be $\mathcal{C}_{\omega\rho}\left(\lambda_l(\phi), \{\lambda_{\hat{\omega}}(\phi)\}_{\hat{\omega} \in \hat{\Omega}^{\rho}}\right)$. Minimizing cost across all techniques, j's shadow unit cost is

$$\min_{\rho \in \varrho(\omega)} \min_{\phi \in \Phi_{\omega j \rho}} \mathcal{C}_{\omega\rho}\left(\lambda_l(\phi), \{\lambda_{\hat{\omega}}(\phi)\}_{\hat{\omega} \in \hat{\Omega}^{\rho}}\right)$$

One implication is that j's total shadow cost can be expressed as

$$c_j y_j = \frac{1}{t_l} w l_j + \sum_{\hat{\omega} \in \Omega_{\rho}} \frac{1}{t_{s_{\hat{\omega}}(\phi)}} p_{s_{\hat{\omega}}(\phi)} x_{s_{\hat{\omega}}(\phi)}$$

We assume that prices in firm-to-firm trade are set at the supplier's shadow unit cost, and that sales to final consumers are determined via monopolistic competition, so that prices are a fixed markup $\frac{\varepsilon}{\varepsilon - 1}$ over shadow unit cost.

While we have no microfoundation for why the wedges would take this particular form, it still may be interesting to explore how our results would differ under this alternative formulation.

A key difference from the baseline model is that the wedges themselves represent behavioral distortion but do not use up resources. To solve for the actual allocation, we need to solve for the resources that get used in producing each good. Towards this, define the "resource gap" for firm j, $a_j \in [0, 1]$, to be the ratio of the labor used across all stages of production (by firm j, its suppliers, its suppliers' suppliers, etc.) to produce a unit of good j and the shadow unit cost of firm j. In other words, if c_j is j's shadow unit cost, $a_j c_j$ is the cumulative expenditure on primary inputs (i.e., labor) to produce a unit of good j. Suppose that j uses technique ϕ, and it produces y_j units of output using labor l_j and $x_{s_{\hat{\omega}}(\phi)}$ units of intermediate inputs used from respective suppliers $s_{\hat{\omega}}(\phi)$. If the firm faces wedges t_l and $\left\{t_{s_{\hat{\omega}}(\phi)}\right\}$, then its resource gap satisfies the equation

$$a_j c_j y_j = w l_j + \sum_{\hat{\omega} \in \Omega_\rho} a_{s_{\hat{\omega}}(\phi)} c_{s_{\hat{\omega}}(\phi)} x_{s_{\hat{\omega}}(\phi)} \tag{14}$$

That is, cumulative expenditure on labor to make good j is equal to the sum of the labor directly used plus the cumulative labor used to make each input.

We briefly summarize the key equations that can be used to estimate the distortions and recipe technologies in this alternative environment, as well as to compute the counterfactual change in aggregate productivity that would result from changes in distortions in this alternative environment. Proofs are relegated to Appendix H.2 below.

Among firms in industry ω that, in equilibrium, use recipe ρ, the following moment conditions hold

$$\begin{aligned} E\left[\frac{s_{Rj}}{\frac{1}{\bar{t}_x}\alpha_R^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \\ E\left[\frac{s_{Lj}}{\frac{1}{t_l}\alpha_L^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \end{aligned} \tag{15}$$

where $\bar{t}_x$ is the harmonic average of the wedges that prevail in equilibrium, as in our baseline environment. The first moment condition is similar to the one that arises in our baseline environment: the expenditure on relationship-specific inputs is shaded down by distortions relative to what the expenditure would be in our baseline environment. The second moment condition differs because, in contrast to our baseline, here the distortions on relationship-specific intermediate inputs do not cause the firm to use extra labor. Following the algorithm outlined in Section 4, these two moment conditions can be used to estimate distortions for each state, $\bar{t}_x$, and technology parameters for each recipe.

We next turn to counterfactuals. As with Section 4, we are interested in the change in aggregate productivity that would come from a change in the distribution of distortions, $T(\cdot)$. Like in Proposition 4, we can solve for this in changes. The change in welfare is

$$\frac{U'}{U} = \frac{\left\{\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\frac{\sum_\omega HH_\omega \bar{a}'_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}}{\sum_\omega HH_\omega \bar{a}_\omega}} \tag{16}$$

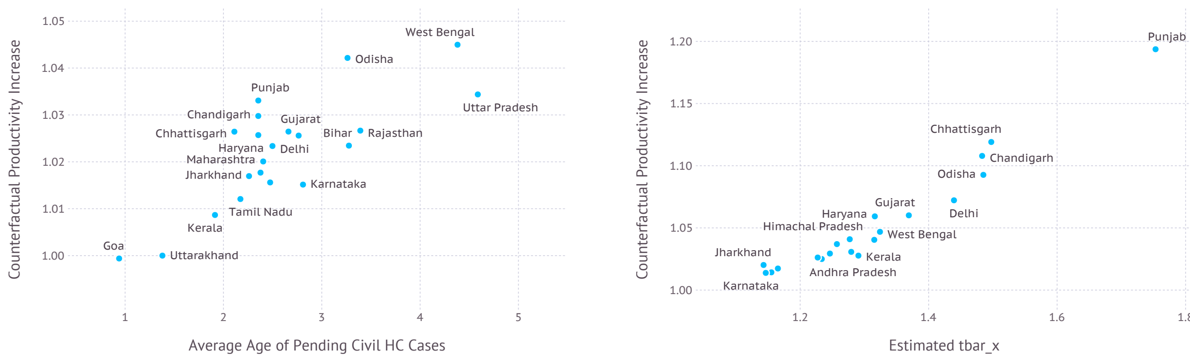
and the change in the shadow cost index for industry ω is

$$\left(\frac{C'_\omega}{C_\omega}\right)^{-\gamma} = \left\{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho} \left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}\right\}^{-\gamma}$$

where HH_ω is the share of the household's expenditure spent on goods in industry ω in the current equilibrium, and $R_{\omega\rho}$ is the share of revenue in industry ω accounted for by firms that use recipe ρ in the current equilibrium. To find the change in aggregate productivity, we need to solve for two extra sets of variables, $\bar{a}_\omega$ and $\bar{a}'_\omega$, the average resource gap for each industry in the current equilibrium and in the counterfactual. These can be solved for recursively with the following two equations

$$\bar{a}_\omega = \sum_{\rho \in \varrho_\omega} R_{\omega\rho} \left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega} \in \hat{\Omega}_\rho^R} \frac{1}{\bar{t}_x} \bar{a}_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega} \in \hat{\Omega}_\rho^H} \bar{a}_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho\right\}$$

$$\bar{a}'_\omega = \sum_{\rho \in \varrho_\omega} R_{\omega\rho} \left(\frac{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho} \left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}}{C'_\omega / C_\omega}\right)^{-\gamma} \left\{\frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega} \in \hat{\Omega}_\rho^R} \frac{1}{\bar{t}'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega} \in \hat{\Omega}_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho\right\}$$



(a) Improving court speed to level of best court

(b) Halving wedges

Figure H.1 Counterfactual increases in aggregate productivity, alternative model

The figure shows the counterfactual increase in U when the wedges on relationship-specific inputs are reduced in the alternative model where distortions do not entail a resource cost. In the left panel we reduce $\bar{t}_x$ according to the fraction of $\bar{t}_x$ that is explained by court congestion in a linear IV regression; in the right panel we cut the $\bar{t}_x$ in half.

Provided that $\alpha_L^\rho > 0$ in each industry, each of these equations is a contraction. Note that relative to the baseline model in the main text of the paper, no extra information is required to estimate the model or compute counterfactuals.

H.1 Results for Alternative Formulation of the Distortion

Figure H.1 shows the results from conducting the counterfactual (Equation 16) using the parameter estimates from the moment conditions (15). Counterfactual welfare changes are a bit smaller than in the benchmark model, where a reduction in the distortions also frees up labor that can be used in production.

H.2 Proofs for Alternative Formulation of the Distortion

Let $F_\omega(c)$ be the fraction of firms in industry ω with shadow unit cost weakly less than c. As in the baseline economy, Proposition 1 applies, so that $F_\omega(c) = 1 - e^{-(c/C_\omega)^\gamma}$ where the shadow cost indices for each industry $\{C_\omega\}$ satisfy

$$\begin{aligned} C_\omega &= \left\{ \sum_{\rho \in \varrho_\omega} \kappa_{\omega\rho} B_{\omega\rho} \left((t_x^*)^{\alpha_R^\rho} t_l^{\alpha_L^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} \\ t_x^* &= \left(\int_1^\infty t_x^{-\zeta_R} dT(t_x) \right)^{-1/\zeta_R} \end{aligned} \tag{17}$$

and $\kappa_{\omega\rho}$ is a constant that depends on technological parameters.

We begin by deriving two results that will be helpful in characterizing the equilibrium. We also define $\tilde{F}_\omega(c, a)$ to be the fraction of firms in industry ω with shadow unit cost weakly less than c and resource gap weakly less than a.

Lemma H.1 *Among suppliers of any input that are selected in equilibrium, the effective shadow cost λ delivered by the suppler, resource gap of the supplier, a, and the wedge facing the buyer of using that supplier, t_x, are mutually independent.*

Proof. Consider a technique of recipe ρ for which the common component of productivity is $b_{\hat{\omega}}(\phi)$. For any such technique, the arrival rate of suppliers with resource gap less than a, for which the wedge facing the buyer would be less that t_0 and that delivers effective shadow cost weakly less than λ (which means that if match-specific productivity is z, the supplier's shadow cost c is small enough so that $\lambda \leq \frac{tc}{zb_{\hat{\omega}}(\phi)}$, i.e., $\frac{\lambda z b_{\hat{\omega}}(\phi)}{t} \leq c$) is

$$\begin{aligned}
\int_0^\infty \int_1^{t_0} \tilde{F}_{\hat{\omega}}\left(\frac{\lambda z b_{\hat{\omega}}(\phi)}{t}, a\right) dT(t)\zeta_{\hat{\omega}} z^{-\zeta_{\hat{\omega}}-1} dz &= [\lambda b_{\hat{\omega}}(\phi)]^{\zeta_{\hat{\omega}}} \int_0^\infty \int_1^{t_0} \tilde{F}_{\hat{\omega}}(u,a) t^{-\zeta_{\hat{\omega}}} dT(t) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du \\
&= [\lambda b_{\hat{\omega}}(\phi)]^{\zeta_{\hat{\omega}}} \int_1^{t_0} t^{-\zeta_{\hat{\omega}}} dT(t) \int_0^\infty \tilde{F}_{\hat{\omega}}(u,a) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du \\
&= \left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \frac{\int_1^{t_0} t^{-\zeta_{\hat{\omega}}} dT(t)}{\int_1^\infty t^{-\zeta_{\hat{\omega}}} dT(t)} \frac{\int_0^\infty \tilde{F}_{\hat{\omega}}(u,a) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du}{\int_0^\infty F_{\hat{\omega}}(u) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du} \\
&= \left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \tilde{T}(t_0) A_{\hat{\omega}}(a)
\end{aligned}$$

where $\tilde{T}(t_0) \equiv \frac{\int_1^{t_0} t^{-\zeta_{\hat{\omega}}} dT(t)}{\int_1^\infty t^{-\zeta_{\hat{\omega}}} dT(t)}$ and $\Lambda_{\hat{\omega}} \equiv \begin{cases} t_x^* \left[\int_0^\infty c^{-\zeta_R} dF_{\hat{\omega}}(c)\right]^{-1/\zeta_R}, & \hat{\omega} \in \Omega_R^\rho \\ \left[\int_0^\infty c^{-\zeta_H} dF_{\hat{\omega}}(q)\right]^{-1/\zeta_H}, & \hat{\omega} \in \Omega_H^\rho \end{cases}$ are defined as in the baseline economy and $A_{\hat{\omega}}(a) \equiv \frac{\int_0^\infty \tilde{F}_{\hat{\omega}}(u,a)\zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du}{\int_0^\infty F_{\hat{\omega}}(u)\zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du}$. We can differentiate to find the arrival rate of suppliers that deliver effective cost λ and resource gap weakly less than v and with wedge weakly less than t_0,

$$\zeta_{\hat{\omega}} \lambda^{\zeta_{\hat{\omega}}-1} \left[\frac{b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \tilde{T}(t_0) A_{\hat{\omega}}(a)$$

Next note that arrival rate of suppliers that deliver effective shadow cost weakly less than λ is $\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}$, so the probability that no such techniques arrive is $e^{-\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}}$. Together, to find the probability that the best supplier of an input for a technique delivers effective cost weakly less than λ_0, has resource gap weakly less than a, and comes with a wedge for the buyer of t_0, we simply integrate over possible value of $\lambda \in [0, \lambda_0]$ the arrival rate of a supplier with these properties multiplied by the probability that there is no better supplier (these are indepenent events)

$$\begin{aligned}
\int_0^{\lambda_0} e^{-\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}} \zeta_{\hat{\omega}} \lambda^{\zeta_{\hat{\omega}}-1} \left[\frac{b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \tilde{T}(t_0) A_{\hat{\omega}}(a) d\lambda &= A_{\hat{\omega}}(a) \tilde{T}(t_0) \int_0^{\lambda_0} \zeta_{\hat{\omega}} \lambda^{\zeta_{\hat{\omega}}-1} \left[\frac{b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} e^{-\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}} d\lambda \\
&= A_{\hat{\omega}}(a) \tilde{T}(t_0) e^{-\left[\frac{\lambda_0 b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}}
\end{aligned}$$

Since the resource gap, a, the wedge facing the buyer, t_0, and the effective cost λ_0 are multiplicatively separable, they are mutually independent. Finally, since only λ_0 enters the probability that the technique is actually used by the buyer, it must be that the three are mutually independent across suppliers of the input that are actually used in equilibrium. ∎

Lemma H.2 *Among firms in any industry, shadow cost c and resource gap a are independent.*

Proof. For a firm in industry ω, let $\tilde{H}_{\omega\rho}(c,a)$ be arrival rate of techniques with recipe ρ that delivers shadow cost weakly less than c and resource wedge weakly less than a for the buyer.

Consider a single technique of recipe ρ that uses labor and n intermediate inputs, for which the common components of input-augmenting productivities are $b = \{b_1, b_1, ..., b_n\}$. The probability that the technique delivers a shadow cost weakly less than c and resource gap weakly less than a_0 is

$$\int_0^\infty ... \int_0^\infty 1\left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}\left(\lambda_l,\lambda_1,...,\lambda_n\right)\leq c, \\ \frac{1}{t_l}\frac{\lambda_l\mathcal{C}_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}+\frac{a_1}{t_1}\frac{\lambda_1\mathcal{C}_{\omega\rho\hat{\omega}_1}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}+...+\frac{a_n}{t_n}\frac{\lambda_n\mathcal{C}_{\omega\rho\hat{\omega}_n}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\leq a_0 \end{array}\right\}$$
$$\times\prod_{k=1}^n A_{\hat{\omega}}\left(a_k\right)T_{\hat{\omega}}\left(t_k\right)e^{-\left(\frac{\lambda_k b_k}{\Lambda_{\hat{\omega}_k}}\right)^{\zeta_{\hat{\omega}}}}\frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}}\zeta_{\hat{\omega}_k}\lambda_k^{\zeta_{\hat{\omega}_k}-1}d\lambda_{\hat{\omega}_k}$$

where $\lambda_l=\frac{t_l}{b_l}$. To find $\tilde{H}_{\omega\rho}(c,a_0)$, we simply need to integrate over the arrival of such techniques across realizations of the vector b:

$$\begin{aligned}\tilde{H}\left(c,a_0\right) &= \int_0^\infty ... \int_0^\infty 1\left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}\left(\lambda_l,\lambda_1,...,\lambda_n\right)\leq c, \\ \frac{1}{t_l}\frac{\lambda_l\mathcal{C}_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}+\frac{a_1}{t_1}\frac{\lambda_1\mathcal{C}_{\omega\rho\hat{\omega}_1}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}+...+\frac{a_n}{t_n}\frac{\lambda_n\mathcal{C}_{\omega\rho\hat{\omega}_n}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\leq a_0 \end{array}\right\} \\ &\quad\times\prod_{k=1}^n A_{\hat{\omega}}\left(da_k\right)T_{\hat{\omega}}\left(dt_k\right)e^{-\left(\frac{\lambda_k b_k}{\Lambda_{\hat{\omega}_k}}\right)^{\zeta_{\hat{\omega}}}}\frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}}\zeta_{\hat{\omega}_k}\lambda_k^{\zeta_{\hat{\omega}_k}-1}d\lambda_{\hat{\omega}_k}\mathcal{B}_{\omega\rho}\left(db\right)\end{aligned}$$

Using the definition of $\lambda_l=\frac{t_l}{b_l}$ and the homogeneity of the cost function, this is

$$\begin{aligned}\tilde{H}\left(c,a_0\right) &= \int_0^\infty ... \int_0^\infty 1\left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l c},\frac{\lambda_1}{c},...,\frac{\lambda_n}{c}\right)\leq 1, \\ \frac{1}{t_l}\frac{\lambda_l}{c}\mathcal{C}_{\omega\rho l}\left(\frac{\lambda}{c}\right)+\frac{a_1}{t_1}\frac{\lambda_1}{c}\mathcal{C}_{\omega\rho\hat{\omega}_1}\left(\frac{\lambda}{c}\right)+...+\frac{a_n}{t_n}\frac{\lambda_n}{c}\mathcal{C}_{\omega\rho\hat{\omega}_n}\left(\frac{\lambda}{c}\right)\leq a_0 \end{array}\right\} \\ &\quad\times\prod_{k=1}^n A_{\hat{\omega}}\left(da_k\right)T_{\hat{\omega}}\left(dt_k\right)e^{-\left(\frac{\lambda_k b_k}{\Lambda_{\hat{\omega}_k}}\right)^{\zeta_{\hat{\omega}}}}\frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}}\zeta_{\hat{\omega}_k}\lambda_k^{\zeta_{\hat{\omega}_k}-1}d\lambda_{\hat{\omega}_k}\mathcal{B}_{\omega\rho}(db)\end{aligned}$$

It will be useful to make the change of variables $v_k=\frac{\lambda_k}{c}$, $v_l=\frac{t_l}{cb_l}$, and $m_j=\left(\frac{\lambda_l b_l}{\Lambda_{\hat{\omega}}}\right)^{\zeta_{\hat{\omega}_k}}$ to express $\tilde{H}\left(c,a\right)$ as

$$\begin{aligned}\tilde{H}\left(c,a_0\right) &= \int_0^\infty ... \int_0^\infty 1\left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}\left(v_l,v_1,...,v_n\right)\leq 1, \\ \frac{1}{t_l}v_l\mathcal{C}_{\omega\rho l}\left(v\right)+\frac{a_1}{t_1}v_1\mathcal{C}_{\omega\rho\hat{\omega}_1}\left(v\right)+...+\frac{a_n}{t_n}v_n\mathcal{C}_{\omega\rho\hat{\omega}_n}\left(v\right)\leq a_0 \end{array}\right\} \\ &\quad\times B_{\omega\rho}\left(\frac{t_l}{c}\right)^{-\beta_l^\rho}\prod_{k=1}^n A_{\hat{\omega}}\left(da_k\right)T_{\hat{\omega}}\left(dt_k\right)\left(\frac{m_k^{1/\zeta_{\hat{\omega}_k}}\Lambda_\omega}{c}\right)^{-\beta_{\hat{\omega}_k}}e^{-m_k}dm_k\mathcal{V}\left(dv\right)\end{aligned}$$

or more simply

$$\tilde{H}\left(c,a_0\right)=\bar{A}_{\omega\rho}\left(a_0\right)c^\gamma$$

where

$$\begin{aligned}\bar{A}_{\omega\rho}\left(a_0\right) &\equiv B_{\omega\rho}t_l^{-\beta_l^\rho}\Lambda_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho}...\Lambda_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho}\int_0^\infty ... \int_0^\infty 1\left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}\left(v_l,v_1,...,v_n\right)\leq 1, \\ \frac{1}{t_l}v_l\mathcal{C}_{\omega\rho l}\left(v\right)+\frac{a_1}{t_1}v_1\mathcal{C}_{\omega\rho\hat{\omega}_1}\left(v\right)+...+\frac{a_n}{t_n}v_n\mathcal{C}_{\omega\rho\hat{\omega}_n}\left(v\right)\leq a_0 \end{array}\right\} \\ &\quad\times\prod_{k=1}^n A_{\hat{\omega}}\left(da_k\right)T_{\hat{\omega}}\left(dt_k\right)m_k^{-\beta_{\hat{\omega}_k}/\zeta_{\hat{\omega}_k}}e^{-m_k}dm_k\mathcal{V}\left(dv\right)\end{aligned}$$

We can differentiate to find the arrival rate of techniques that deliver shadow cost c and resource cost no greater than a

$$\bar{A}_{\omega\rho}\left(a\right)\gamma c^{\gamma-1}$$

Next note that arrival rate of techniques of recipe ρ that deliver shadow cost weakly less than c regardless of resource gap is $\bar{A}_{\omega\rho}(1)c^\gamma$, so that the probability of no such techniques across all recipes is $\prod_{\rho\in\varrho_\omega}e^{-\bar{A}_{\omega\rho}(1)c^\gamma}=e^{-c^\gamma\sum_{\rho\in\varrho_\omega}\bar{A}_{\omega\rho}(1)}=e^{-(c/C_\omega)^\gamma}$ (which follows from the definition of C_ω). Together, to find the probability that the best technique delivers shadow cost weakly less than c_0 and has resource gap weakly less than a, we simply integrate over possible values of $c\in[0,c_0]$ the arrival rate of a supplier with these properties multiplied by the probability that there is no better supplier (these are independent events)

$$\begin{aligned}\tilde{F}(c,a) &= \sum_{\rho\in\varrho_\omega}\int_0^{c_0} e^{-(c/C_\omega)^\gamma}\bar{A}_{\omega\rho}(a)\gamma c^{\gamma-1}dc \\ &= \sum_{\rho\in\varrho_\omega}\bar{A}_{\omega\rho}(a)\int_0^{c_0} e^{-(c/C_\omega)^\gamma}\gamma c^{\gamma-1}dc \\ &= \frac{\sum_{\rho\in\varrho_\omega}\bar{A}_{\omega\rho}(a)}{C_\omega^{-\gamma}}e^{-(c_0/C_\omega)^\gamma}\end{aligned}$$

The result follows from the fact that this is multiplicatively separable in c_0 and a. ■

Claim H.1 *Among firms in industry ω that choose to use recipe ρ, the following two equations hold*

$$\begin{aligned}E\left[\frac{s_{Rj}}{\frac{1}{t_x}\alpha_R^\rho}-\frac{s_{Hj}}{\alpha_H^\rho}\middle|\rho\right] &= 0 \\ E\left[\frac{s_{Lj}}{\frac{1}{t_l}\alpha_L^\rho}-\frac{s_{Hj}}{\alpha_H^\rho}\middle|\rho\right] &= 0\end{aligned}$$

Proof. Let p_s be the actual price per unit paid to supplier so that $t_s p_s$ is the shadow cost per unit of the input to the buyer. The share of j's revenue paid to the supplier of input $\hat{\omega}$ can be expressed as

$$s_{\hat{\omega}j}=\frac{p_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{\text{Rev}_j}=\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{t_{s_{\hat{\omega}}(\phi)}p_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{c_jy_j}\frac{c_jy_j}{\text{Rev}_j}=\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{c_jy_j}{\text{Rev}_j}$$

where the last equality follows from Shephard's lemma. Similarly, the share of revenue spent on labor is

$$s_{Lj}=\frac{wl_j}{\text{Rev}_j}=\frac{1}{t_l}\frac{t_lwl_j}{c_jy_j}\frac{c_jy_j}{\text{Rev}_j}=\frac{1}{t_l}\frac{\lambda_L\mathcal{C}_{\omega\rho L}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{c_jy_j}{\text{Rev}_j}$$

Among firms that produce using recipe ρ whose shadow cost is c, the average share of revenue spent on relationship-specific inputs is

$$E[s_{Rj}|c_j,\rho]=\sum_{\hat{\omega}\in\Omega_\rho^R}E[s_{\hat{\omega}j}|c_j,\rho]=\sum_{\hat{\omega}\in\Omega_\rho^R}E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\middle|c,\rho\right]$$

Using the law of iterated expectations along with $E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\middle|\lambda\right]=\frac{1}{t_x}$, this is

$$\begin{aligned}E[s_{Rj}|c_j,\rho] &= \sum_{\hat{\omega}\in\Omega_\rho^R}E\left[E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\middle|\lambda,c,\rho\right]\middle|c,\rho\right] \\ &= \sum_{\hat{\omega}\in\Omega_\rho^R}E\left[E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\middle|\lambda,c,\rho\right]\frac{\lambda_{s_{\hat{\omega}}(\phi)}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\middle|c,\rho\right] \\ &= \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{t_x}E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\middle|c,\rho\right]\end{aligned}$$

Finally, note that, conditional on shadow cost c, downstream demand and prices do not depend on any of the determinants of c. This implies that for a firm in industry ω, $E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\middle|c,\rho\right]=E\left[\frac{cy}{\text{Rev}}\middle|c,\rho\right]E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle|c,\rho\right]$. Using Proposition F.2, we have

$$E[s_{Rj}|c_j,\rho]=\sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{t_x}\alpha_{\hat{\omega}}^\rho E\left[\frac{cy}{\text{Rev}}|c,\rho\right]=\frac{1}{t_x}\alpha_R^\rho E\left[\frac{cy}{\text{Rev}}|c,\rho\right]$$

Using similar logic, we have

$$
\begin{aligned}
E\left[s_{Hj}|c_j\right] &= \sum_{\hat{\omega}\in\Omega_\rho^H} E\left[s_{\hat{\omega}j}|c_j\right] = \sum_{\hat{\omega}\in\Omega_\rho^H} E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}\left(\lambda\right)}{\mathcal{C}_{\omega\rho}\left(\lambda\right)}\frac{cy}{\text{Rev}}\middle| c\right] = \sum_{\hat{\omega}\in\Omega_\rho^H} \alpha_{\hat{\omega}}^\rho E\left[\frac{cy}{\text{Rev}}\middle| c\right] = \alpha_H^\rho E\left[\frac{cy}{\text{Rev}}\middle| c\right] \\
E\left[s_{Lj}|c_j\right] &= E\left[\frac{1}{t_l}\frac{\lambda_l\mathcal{C}_{\omega\rho L}\left(\lambda\right)}{\mathcal{C}_{\omega\rho}\left(\lambda\right)}\frac{cy}{\text{Rev}}\middle| c\right] = \frac{1}{t_l}\alpha_L^\rho E\left[\frac{cy}{\text{Rev}}\middle| c\right]
\end{aligned}
$$

Elminating $E\left[\frac{cy}{\text{Rev}}\middle| c\right]$, we have the two moment conditions

$$
\begin{aligned}
E\left[\frac{s_{Rj}}{\frac{1}{t_x}\alpha_R^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \\
E\left[\frac{s_{Lj}}{\frac{1}{t_l}\alpha_L^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0
\end{aligned}
$$

■

Lemma H.3 *Let a_ω be the average resource gap among firms in industry ω. Then*

$$
\bar{a}_\omega = \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}
$$

Proof. We begin by dividing (14) by the total shadow cost $c_j y_j$ and rearranging.

$$
\begin{aligned}
a_j &= \frac{wl_j}{c_j y_j} + \sum_{\hat{\omega}\in\Omega_\rho} a_{s_{\hat{\omega}}(\phi)}\frac{c_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{c_j y_j} \\
&= \frac{1}{t_l}\frac{t_l w l_j}{c_j y_j} + \sum_{\hat{\omega}\in\Omega_\rho}\frac{a_{s_{\hat{\omega}}(\phi)}}{t_{s_{\hat{\omega}}(\phi)}}\frac{t_{s_{\hat{\omega}}(\phi)}c_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{c_j y_j} \\
&= \frac{1}{t_l}\frac{\lambda_{jl}\mathcal{C}_{\omega\rho l}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)} + \sum_{\hat{\omega}\in\Omega_\rho}\frac{a_{s_{\hat{\omega}}(\phi)}}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)}
\end{aligned}
$$

Among firms in industry ω that use recipe ρ and have a vector of effective shadow cost λ, the average resource gap is

$$
\begin{aligned}
E\left[a_j|\lambda,\omega,\rho\right] &= \frac{1}{t_l}\frac{\lambda_{jl}\mathcal{C}_{\omega\rho l}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)} + \sum_{\hat{\omega}\in\Omega_\rho} E\left[\frac{a_{s_{\hat{\omega}}(\phi)}}{t_{s_{\hat{\omega}}(\phi)}}|\omega,\rho\right]\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)} \\
&= \frac{1}{t_l}\frac{\lambda_{jl}\mathcal{C}_{\omega\rho l}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)} + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)} + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}\left(\lambda_j\right)}{\mathcal{C}_{\omega\rho}\left(\lambda_j\right)}
\end{aligned}
$$

Then the average resource gap across all firms that use ρ is

$$
E\left[a_j|\omega,\rho\right] = E\left[E\left[a_j|\lambda,\omega,\rho\right]|\omega,\rho\right] = \frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{v}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho
$$

To find the average resource gap, we simply weight the previous equation by the probability that a firm chooses to use recipe ρ, which, in equilibrium, is equal to the share of revenue earned by firms that use recipe ρ.

$$
\bar{a}_\omega = \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}
$$

■

Claim H.2 *Suppose that the distribution of wedges changes from T to T'. The following equations are sufficient to compute the counterfactual change in aggregate productivity:*

$$\begin{aligned}
\frac{U'}{U} &= \frac{\left\{\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\frac{\sum_\omega HH_\omega \bar{a}'_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}}{\sum_\omega HH_\omega \bar{a}_\omega}} \\
\left(\frac{C'_\omega}{C_\omega}\right)^{-\gamma} &= \left\{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho}\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\Omega_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}\right\}^{-\gamma} \\
\bar{a}_\omega &= \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\} \\
\bar{a}'_\omega &= \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left(\frac{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho}\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\Omega_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}}{C'_\omega/C_\omega}\right)^{-\gamma}\left\{\frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}'_x}\bar{a}'_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}'_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}
\end{aligned}$$

Proof. As in our baseline, the price level in industry ω and in aggregate are respectively

$$\begin{aligned}
P_\omega &= \frac{\varepsilon}{\varepsilon-1}J_\omega^{\frac{1}{1-\varepsilon}}\Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{1}{1-\varepsilon}}C_\omega \\
P &= \frac{\varepsilon}{\varepsilon-1}\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}}\Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}}C_\omega^{1-\eta}\right\}^{\frac{1}{1-\eta}}
\end{aligned}$$

Total profit generated by the production of good j (revenue from final consumer less cumulative labor cost) is $(p_j - a_j c_j)\,u_j$. Using $c_j = \frac{\varepsilon-1}{\varepsilon}p_j$ and $u_j = U\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon}$, gives

$$\begin{aligned}
(p_j - a_j c_j)\,u_j &= \left(p_j - a_j\frac{\varepsilon-1}{\varepsilon}p_j\right)U\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon} \\
&= \left(1 - a_j\frac{\varepsilon-1}{\varepsilon}\right)UP\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}
\end{aligned}$$

Summing across all firms in the economy and using the fact that v_j is independent of p_j gives

$$\begin{aligned}
\Pi &= \sum_\omega\int_0^{J_\omega}\left(1 - a_j\frac{\varepsilon-1}{\varepsilon}\right)UP\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}dj \\
&= UP - \frac{\varepsilon-1}{\varepsilon}U\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta}\int_0^{J_\omega}a_j\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}dj \\
&= UP - \frac{\varepsilon-1}{\varepsilon}U\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta}\bar{a}_\omega\int_0^{J_\omega}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}dj \\
&= UP - \frac{\varepsilon-1}{\varepsilon}U\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta}\bar{a}_\omega
\end{aligned}$$

Plugging this into the household's budget constraint gives

$$\begin{aligned} UP &= wL + \Pi \\ &= wL + UP - \frac{\varepsilon - 1}{\varepsilon} U \sum_{\omega} \upsilon_\omega P^\eta \left(P_\omega\right)^{1-\eta} \bar{a}_\omega \end{aligned}$$

Rearranging and using the expression for and using the expressions for P_ω and P gives

$$\begin{aligned} U &= \frac{w}{\frac{\varepsilon-1}{\varepsilon} \sum_\omega \upsilon_\omega P^\eta \left(P_\omega\right)^{1-\eta} a_\omega} L \\ &= \Gamma \left(1 - \frac{\varepsilon - 1}{\gamma}\right)^{\frac{1}{\varepsilon-1}} \frac{\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\sum_\omega \bar{a}_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}} L \end{aligned}$$

To find a counterfactual, we thus have

$$\frac{U'}{U} = \frac{\frac{\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \left(C'_\omega\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\sum_\omega \bar{a}'_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \left(C'_\omega\right)^{1-\eta}}}{\frac{\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \left(C_\omega\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\sum_\omega \bar{a}_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \left(C_\omega\right)^{1-\eta}}} = \frac{\left\{\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\frac{\sum_\omega HH_\omega \bar{a}'_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}}{\sum_\omega HH_\omega \bar{a}_\omega}}$$

We thus need expressions for $\bar{a}_\omega$ and $\bar{a}'_\omega$. The expression for $\bar{a}_\omega$ comes directly from Lemma H.3, which also delivers an expression for $\bar{a}'_\omega$:

$$\begin{aligned} \bar{a}'_\omega &= \sum_{\rho \in \varrho_\omega} R'_{\omega\rho} \left\{ \frac{1}{t'_l} \alpha_L^\rho + \sum_{\hat{\omega} \in \Omega_\rho^R} \frac{1}{t'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega} \in \Omega_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho \right\} \\ &= \sum_{\rho \in \varrho_\omega} \frac{R'_{\omega\rho}}{R_{\omega\rho}} R_{\omega\rho} \left\{ \frac{1}{t'_l} \alpha_L^\rho + \sum_{\hat{\omega} \in \Omega_\rho^R} \frac{1}{t'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega} \in \Omega_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho \right\} \\ &= \sum_{\rho \in \varrho_\omega} \frac{R'_{\omega\rho}}{R_{\omega\rho}} R_{\omega\rho} \left\{ \frac{1}{t'_l} \alpha_L^\rho + \sum_{\hat{\omega} \in \Omega_\rho^R} \frac{1}{t'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega} \in \Omega_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho \right\} \end{aligned}$$

Then, noting that among firms in industry ω, the share of revenue of those that use recipe ρ is $R_{\omega\rho} = \kappa_{\omega\rho} B_{\omega\rho} \left(\frac{t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \Omega_\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}}{C_\omega} \right)^{-\gamma}$, the change in revenue share in the counterfactual is

$$\frac{R'_{\omega\rho}}{R_{\omega\rho}} = \left(\frac{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho} \left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \Omega_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}}{C'_\omega / C_\omega} \right)^{-\gamma}$$

The results follows from combining these last two equations. ∎

H.3 A Hsieh-Klenow exercise

In the following, we try to do a quantification exercise that is as close as possible to Hsieh and Klenow (2009), and point out some of the issues we would face.

Suppose that each plant in industry ω that uses recipe ρ uses the production function

$$y_{\omega\rho j} = A_{\omega\rho j} L_{\omega\rho j}^{\alpha_L^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} x_{\omega\rho j\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}$$

The planner maximizes output as

$$u^* = \max_{u_\omega, y_{\omega j}, L_{\omega j}, x_{\omega j\hat{\omega}}} \left(\sum_{\omega\in\Omega} \beta_\omega^{\frac{1}{\eta}} u_\omega^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

subject to

$$\lambda_{\omega\rho j}^* : y_{\omega\rho j} \leq A_{\omega\rho j} L_{\omega\rho j}^{\alpha_L^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} x_{\omega\rho j\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}$$

$$\lambda_\omega^* : u_\omega + \sum_{\omega'} \sum_{\rho\in\varrho_{\omega'}} \sum_{j\in J_{\omega'\rho}} x_{\omega'\rho j\omega} \leq \left(\sum_{\rho\in\varrho_\omega} \sum_{j\in J_{\omega\rho}} y_{\omega\rho j}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$w^* : \sum_{\omega} \sum_{\rho\in\varrho_\omega} \sum_{j\in J_{\omega\rho}} L_{\omega\rho j} \leq L$$

If u is output in the current equilibrium, allocational efficiency is $\frac{u}{u^*}$. We use the following result:

Proposition H.1 *Define $M_\omega \equiv \frac{\lambda_\omega^*/w^*}{p_\omega/w}$. Allocational efficiency is*

$$\frac{u}{u^*} = \frac{pu}{wL} \left(\sum \frac{p_\omega u_\omega}{pu} M_\omega^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

and the $\{M_\omega\}$ solve the system of equations

$$M_\omega = \left\{ \sum_{\rho\in\varrho_\omega} \sum_{j\in J_{\omega\rho}} \frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega} \left[\left(\frac{wL_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}} \right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho} \left(M_{\hat{\omega}} \frac{p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}} \right)^{\alpha_{\hat{\omega}}^\rho} \right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}.$$

Proof. We begin by showing that $u^* = w^* L$. To see this, note that the FOCs for all firms' inputs imply

$$w^* L_\omega^* + \sum_{\rho\in\varrho_\omega} \sum_{\hat{\omega}\in\Omega^\rho} \lambda_{\hat{\omega}}^* x_{\omega\hat{\omega}}^* = \lambda_\omega^* y_\omega^* = \lambda_\omega^* u_\omega^* + \sum_{\omega'} \sum_{\rho\in\varrho_{\omega'}} \lambda_\omega^* x_{\omega'\omega}^*$$

Summing across ω gives and noticing that the terms for intermediate inputs drop gives

$$w^* L = \sum_\omega w^* L_\omega^* = \sum_\omega \lambda_\omega^* u_\omega^*$$

The planner's FOC for u_ω^* is $u_\omega^* = (\lambda_\omega^*)^{-\eta} u^*$, which implies $\left[\sum_\omega (\lambda_\omega^*)^{1-\eta}\right]^{\frac{1}{1-\eta}} = 1$ and hence $\sum_\omega \lambda_\omega^* u_\omega^* = u^*$. The latter implies $w^* L = u^*$

Next, we derive the expression for $\frac{u}{u^*}$, which can be rearranged as $\frac{u}{u^*} = \frac{u}{w^* L} = \frac{u\left(\sum_\omega (\lambda_\omega^*)^{1-\eta}\right)^{\frac{1}{1-\eta}}}{w^* L}$. Using

$\lambda_\omega^* = M_\omega p_\omega \frac{w^*}{w}$, this can be expressed as

$$\begin{aligned}
\frac{u}{u^*} &= \frac{u}{w^* L}\left(\sum_\omega (\lambda_\omega^*)^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \frac{u}{w^* L}\left(\sum_\omega \left(M_\omega p_\omega \frac{w^*}{w}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \frac{pu}{wL}\left(\sum_\omega \left(\frac{p_\omega}{p}\right)^{1-\eta} M_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \frac{pu}{wL}\left(\sum_\omega \frac{p_\omega u_\omega}{pu} M_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}}
\end{aligned}$$

where the last line uses $\left(\frac{p_\omega}{p}\right)^{1-\eta} = \frac{p_\omega u_\omega}{pu}$.

Lastly we derive the system of equations for $\{M_\omega\}$. The first order conditions for $L_{\omega\rho j}$ and $\{x_{\omega\rho j\hat{\omega}}\}$ imply

$$\begin{aligned}
\lambda_{\omega\rho j}^* &= \frac{\left(\frac{w^*}{\alpha_L^\rho}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{\lambda_{\hat{\omega}}^*}{\alpha_{\hat{\omega}}^\rho}\right)^{\alpha_{\hat{\omega}}^\rho}}{A_{\omega\rho j}} \\
&= \frac{1}{y_{\omega\rho j}}\left(\frac{w^* L_{\omega\rho j}}{\alpha_L^\rho}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{\lambda_{\hat{\omega}}^* x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho}\right)^{\alpha_{\hat{\omega}}^\rho} \\
&= p_{\omega\rho j}\left(\frac{w^* L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{\lambda_{\hat{\omega}}^* x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}
\end{aligned}$$

where the second line uses $A_{\omega\rho j} = y_{\omega\rho j}/\left(L_{\omega\rho j}^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho} x_{\omega\rho j\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}\right)$. Using $\lambda_{\hat{\omega}}^* = M_{\hat{\omega}} p_{\hat{\omega}} \frac{w^*}{w}$, this is

$$\begin{aligned}
\lambda_{\omega\rho j}^* &= p_{\omega\rho j}\left(\frac{w^* L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{M_{\hat{\omega}} \frac{w^*}{w} p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho} \\
&= p_{\omega\rho j}\frac{w^*}{w}\left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{M_{\hat{\omega}} \frac{w^*}{w} p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}
\end{aligned}$$

Household demand satisfies $y_{\omega\rho j} = y_\omega \left(\frac{p_{\omega\rho j}}{p_\omega}\right)^{-\varepsilon}$ which implies $p_{\omega\rho j} = p_\omega \left(\frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega}\right)^{\frac{1}{1-\varepsilon}}$. Plugging this in and rearranging gives

$$\frac{\lambda_{\omega\rho j}^*/w^*}{p_\omega/w} = \left(\frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega}\right)^{\frac{1}{1-\varepsilon}} \left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{M_{\hat{\omega}} \frac{w^*}{w} p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}$$

Finally, we have

$$\begin{aligned}
M_\omega &= \frac{\lambda_\omega^*/w^*}{p_\omega/w} = \left\{\sum_{\rho\in\varrho_\omega}\sum_{j\in J_{\omega\rho}}\left(\frac{\lambda_{\omega\rho j}^*/w^*}{p_\omega/w}\right)^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}} \\
&= \left\{\sum_{\rho\in\varrho_\omega}\sum_{j\in J_{\omega\rho}}\frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega}\left[\left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(M_{\hat{\omega}}\frac{p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}
\end{aligned}$$

■

Empirically implementing Theorem H.1 to get robust measures of allocational efficiency is hard. We face a number of issues, mostly relating to small/zero cost shares:

- Most papers on misallocation (in particular Hsieh and Klenow (2009)) winsorize the cost shares before doing the exercise. Since the dispersion of cost shares is crucial to the magnitude of the result, where the winsorizing threshold is set matters a lot for the outcomes (Rotemberg and White (2017) demonstrate this problem very nicely). See below for an illustration of this problem in our setting.[10]
- One decision we have to take is how to interpret firms having a mix of intermediate inputs that is not exactly the same (along the extensive margin) as that of the recipe. We face the same issue in our model, but it is much more important quantitatively in a Hsieh-Klenow exercise because of the how the gains from reallocation depend higher moments of the distribution of distortions.
- It is not clear how to include multi-product plants in this calculation. Here, we would need to know how much of an input $\hat{\omega}$ is being used in the plant's use of recipe ρ. In contrast, to do the counterfactual in the main text of our paper, we need to know only the recipe sales shares of each plant.

In the following, we implement one version of this exercise. We pretend that the economy of each state consists only of firms that correspond to the single-product plants in our data. We choose $\varepsilon = 4$. We winsorize cost shares such that $\frac{p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha^{\rho}_{\hat{\omega}} p_{\omega\rho j} y_{\omega\rho j}}$ is above a particular threshold (and the inverse is above the inverse of the threshold). We do this by adjusting expenditures $p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}$ of j, but not sales of other firms, or final demand.

Figure H.2 shows the distribution of resulting (inverses of) M_ω for the state of Himachal Pradesh, for winsorizing thresholds of 2%, 5%, and 10%. The resulting counterfactual increases in the final consumer's utility aggregate u are 600%, 330%, and 170%. Hence, results depend crucially on the winsorizing thresholds, which are completely arbitrary. In Figure H.3 we show this across states. The figure shows the consumer utility aggregate relative to its counterfactual undistorted one (u/u^* in Theorem H.1) by state and winsorizing threshold, and separately for whether we use variation within recipes or within 5-digit industries. Again the results depend heavily on the winsorizing threshold. Interestingly, u/u^* are relatively similar across states. The welfare gains accrue to a large extent from the extreme cost shares, so that it does not matter much whether one looks at within-recipe or within-industry variation.

[10] Relatedly, how exactly should we do the winsorizing in a way that is model-consistent is not clear: should it be done by changing expenditure (but then it should also change sales of other firms. Which firms?) or sales (but then we would have to change either final consumer purchases or expenditures of other firms?).

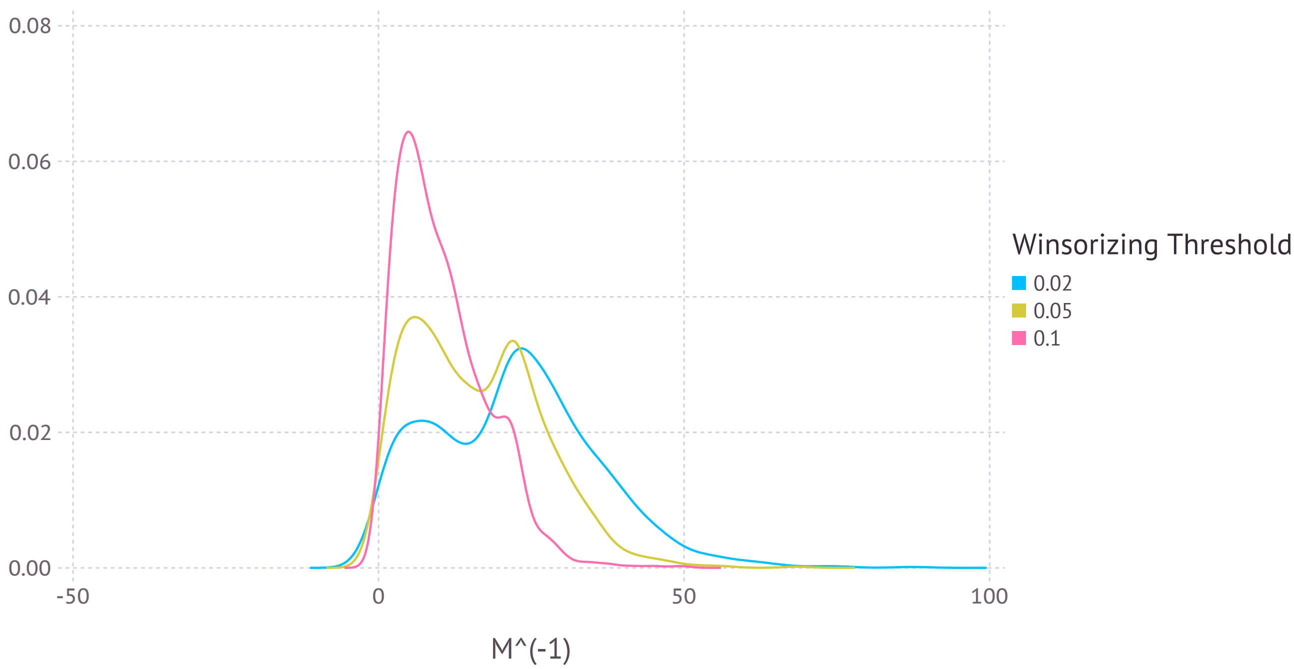
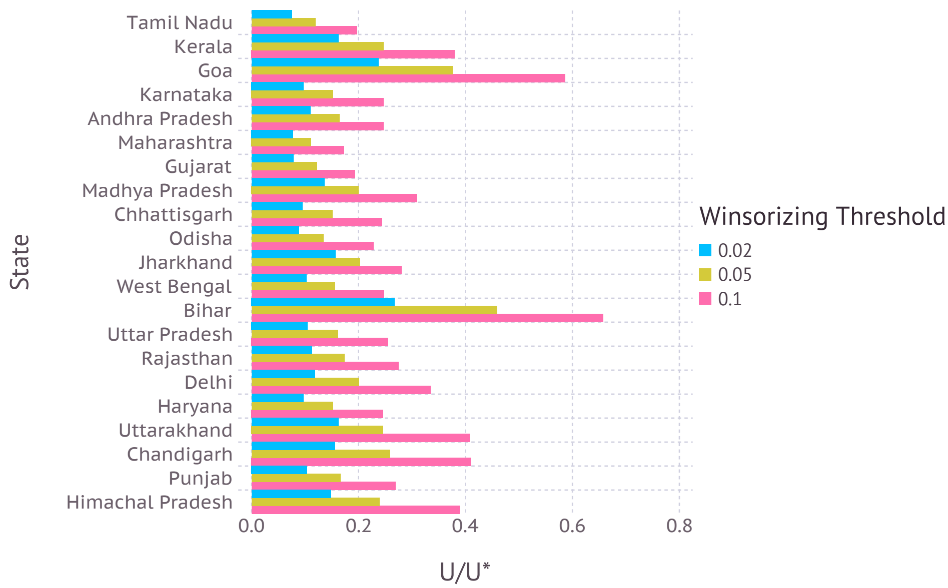
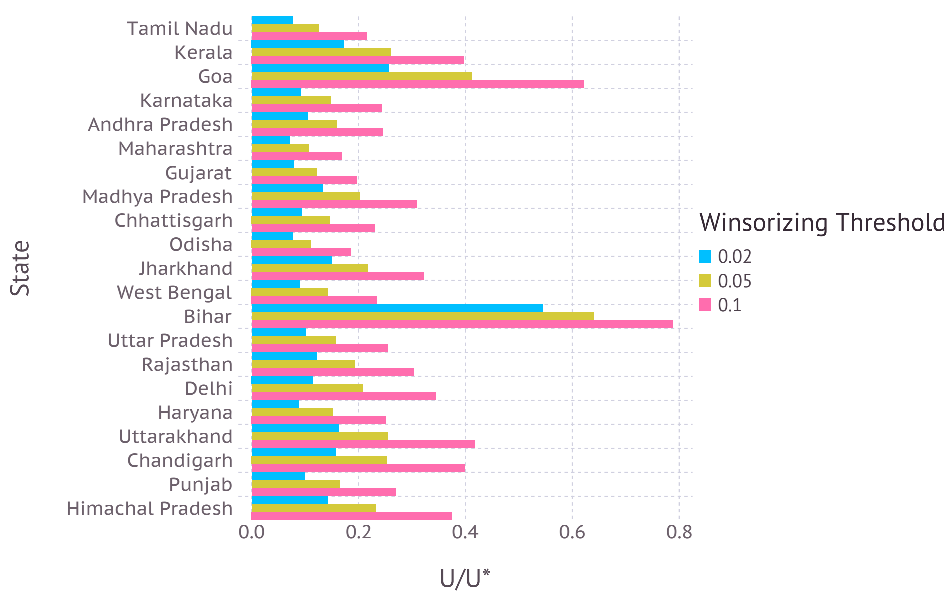


Figure H.2 Counterfactual increases in M_ω, by industry



(a) Within 5-digit Products



(b) Within Recipes

Figure H.3 Hsieh-Klenow Exercise Results, By State

In panel (a), we assume that each product has only one recipe; recipes are therefore equal to products. In panel (b), we use the recipes that emerge from the benchmark results in the main text of the paper.

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